Appendix A

Error magnitude in the conservation of energy in the approximate melt segregation scheme

A.1 Conservation of energy

The approximate melt segregation used in the thermochemical convection models of chapters 6 and 7 has an impact on the conservation of energy, because although 'segregated' melt is added to the system at the top boundary, compaction in the region where the melt is removed is neglected in the numerical model. Here the magnitude of this effect is calculated for a simplified situation to estimate the effect in the convection models. This is done by computing the thermal energy content of an undifferentiated column (case 1, see figure A.1) and comparing it to two differentiated cases (i.e. with a basaltic crust and complementary depleted zone). In the first differentiated case (case 2a, see figure A.1), melt segregation is accompanied by compaction of the melting region. This is the 'true' reference case, in which conservation of both mass and energy are observed. The second differentiated case (case 2b, see figure A.1) is comparable to the result of the approximate melt segregation in scheme in the thermochemical convection models (see section 2.7). Compaction in the melting zone is neglected here, but in spite of this, the produced melt is forced into the top of the model. This results in a certain artificial compaction of the modelled material. This artificial compaction is assumed to be accommodated by the entire domain in a uniform fashion, consistent with test results. The approximations made in the calculations below are:

- the densities of basaltic crust and mantle material are the same;

- the density of mantle material does not change upon depletion;
- the density of any material is independent of temperature and pressure.

The latter two approximations are in line with the (extended) Boussinesq approximation used in the numerical simulations. The result of these assumptions is that partial melting and melt segregation can be regarded as simple transport of material from the melting region to the surface *with conservation of volume*, which simplifies the calculations.

A.2 Thermal energy content

In a 1-D situation the thermal energy content of a domain $[0, z_m]$ is given by:

$$Q = \int_0^{z_m} \rho c_p T dz \tag{A.1}$$

This can be applied to the different cases shown in Figure A.1. The temperature drop ΔT associated with the consumption of latent heat due to partial melting is found from the following expression, which is valid under the conditions listed above:

$$TdS = c_p dT \tag{A.2}$$

$$\Delta T = T_m \cdot (1 - e^{\frac{-F\Delta S}{c_p}}) \tag{A.3}$$

case 1

$$Q_1 = (\frac{1}{2} \cdot z_{tr} + (z_m - z_{tr}))\rho_m c_{pm} T_m$$

$$= (z_m - \frac{1}{2} \cdot z_{tr})\rho_m c_{pm} T_m \tag{A.4}$$

case 2a

$$Q_{2a} = \frac{1}{2} z_{tr} \rho_m c_{pm} T_m + \{z_m - z_{tr} - z_b - (z_{mz} - z_b)\}\rho_m c_{pm} T_m$$

$$+(z_{mz} - z_b)\rho_m c_{pm}(T_m - \Delta T)$$

$$= \frac{1}{2} z_{tr} \rho_m c_{pm} T_m + (z_m - z_{tr} - z_{mz})\rho_m c_{pm} T_m$$

$$+(z_{mz} - z_b)\rho_m c_{pm}(T_m - \Delta T) \tag{A.5}$$

case 2b

$$Q_{2b} = \frac{1}{2} \frac{z_m}{z_m + z_b} z_{tr} \rho_m c_{pm} T_m$$

symbol	parameter	value
ρ_m	density	$3416\mathrm{kgm}^{-3}$
c_{pm}	specific heat	$1250\mathrm{Jkg}^{-1}\mathrm{K}^{-1}$
T_m	mantle temperature	1500 K
F	degree of depletion	
ΔS	entropy of melting	$300\mathrm{Jkg}^{-1}\mathrm{K}^{-1}$

Table A.1: Symbol definitions and parameter values.

$$+\left(z_m - \frac{z_m}{z_m+z_b}z_b - \frac{z_m}{z_m+z_b}z_{tr} - \frac{z_m}{z_m+z_b}z_{mz}\right)\rho_m c_{pm} T_m$$

$$+\frac{z_m}{z_m+z_b}z_{mz}\rho_m c_{pm}(T_m - \Delta T)$$

$$= \frac{1}{2}\frac{z_m}{z_m+z_b}z_{tr}\rho_m c_{pm} T_m + \left(z_m - \frac{z_m}{z_m+z_b}z_b - \frac{z_m}{z_m+z_b}z_{tr}\right)\cdot$$

$$\rho_m c_{pm} T_m - \frac{z_m}{z_m+z_b}z_{mz}\rho_m c_{pm}\Delta T \tag{A.6}$$

A.3 Error magnitude

Using the parameter values listed in Table A.1, we have calculated the difference between heat loss upon differentiation between the scenario including consistent compaction (case 1 → case 2a) and the approximate scenario (case 1 → case 2b), the latter of which is representative of the implementation in the thermochemical convection models of chapters 6 and 7. The results, for three different values of the uniform degree of depletion in the melting zone, are listed in Table A.2. The amount of heat removed from the system by partial melting and cooling of the segregated melt is underestimated by about 10 percent in the approximate melt segregation scheme.

F	ΔQ_{2a}	ΔQ_{2b}	$\frac{\Delta Q_{2b}-\Delta Q_{2a}}{\Delta Q_{2a}}$
(-)	$(\cdot 10^{13}\mathrm{J})$	$(\cdot 10^{13}\mathrm{J})$	(-)
0.1	3.886	3.518	-0.105
0.2	7.606	6.932	-0.097
0.3	11.17	10.24	-0.090

Table A.2: Resulting thermal energy content drops upon differentiation between cases 1 and 2a (ΔQ_{2a}) and 1 and 2b (ΔQ_{2b}), for three different degrees of depletion F (uniform value within the melting zone). The resulting relative errors in the last column show that the approximate scenario of case 2b, comparable to the implementation in the thermochemical convection models of chapters 6 and 7, result in an underestimation of the removal of heat from the system of about 10 percent. Values for the dimensional parameters are: $z_m = 400$ km, z_{tr}= 100 km, z_{mz}=50 km (although the relative results are independent of this value).

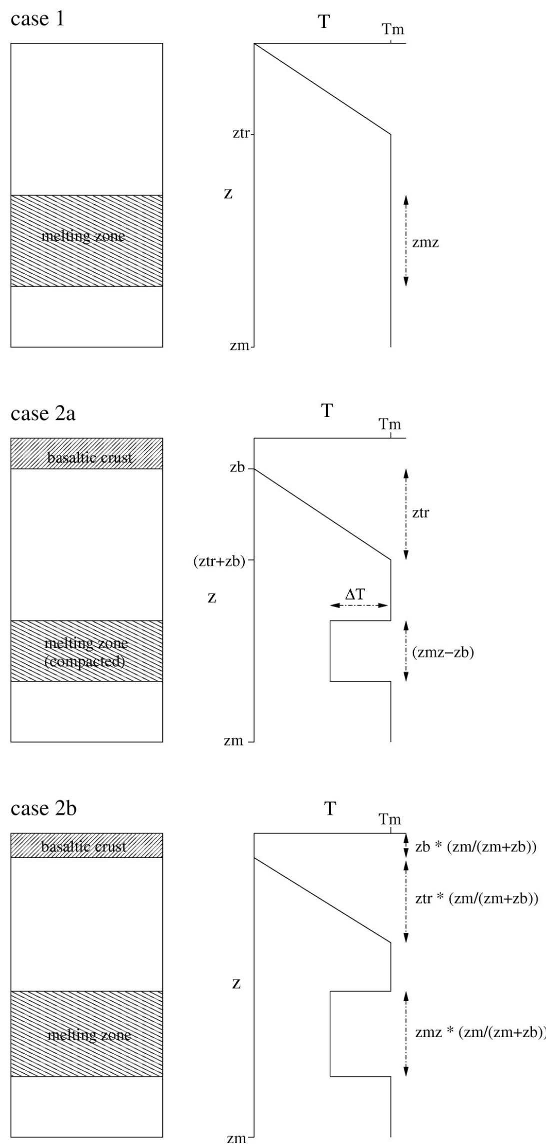

Figure A.1: The geometry (1-D) and geotherms are shown for the three different cases. The first case is before melting. Case 2a is an exact solution after melting. The melting zone compacts as the melt is removed. Case 2b is the approximate solution corresponding to the implementation of the melt segregation process in the numerical models of chapters 6 and 7. No compaction of the melting zone takes place and the compaction required because basaltic material is forced into the domain is accommodated by the entire domain, resulting in prefactors $(z_m/(z_m + z_b))$ in the layer thicknesses.