The crustal structure from teleseismic *P*-wave coda—I. Method

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SUMMARY

The crustal structure beneath a broad-band station has a strong influence on the recorded body wave coda. Conversely, it is evident that crustal receiver effects can be employed to obtain information about the crustal structure at the receiver site. In this paper we present a method to estimate the crustal receiver response, and we invert the response to obtain a model of the most distinct features of the crustal structure beneath the station.

The crustal receiver response of a station is determined from the P-wave coda of deep teleseismic events by deconvolution with an empirically determined source wavelet. This source wavelet is assumed to incorporate source, attenutation and instrument effects. Responses obtained from clusters of events are stacked to enhance the signal-to-noise ratio of the data, and to obtain an estimate of the variance of the data. The crustal P- and S-velocity structure at the station is determined by non-linear waveform inversion of its crustal responses. An inversion strategy was developed to search efficiently for the global minimum in the misfit between data and synthetics. The inversion appears to be most sensitive to large velocity constrasts in the velocity structure which give rise to high-amplitude reflected and converted phases.

The method is illustrated by the inversion of the crustal responses of NARS station NE05. It is shown that a thin layer (1.2 km) of unconsolidated sediments and a strong mid-crustal discontinuity at 11 km depth constitute the most prominent features of the crustal structure at this station. They cause an apparent delay of the first arrival on the radial component of the seismograms and produce high amplitude SV-multiples in the *P*-wave coda.

Key words: body wave modelling, crustal structure, inversion.

INTRODUCTION

Information about the crustal structure beneath the recording station is important for studies that involve a detailed interpretation of body wave data. Without appropriate knowledge of the site effects, it may be impossible to interpret unambiguously the seismic signal in terms of fine earth structure (see e.g. Lay & Helmberger 1983; Silver & Chan 1986; Cummins & Johnson 1988; Weber & Davis 1990). Unfortunately, crustal effects are still insufficiently known for many broad-band stations, thus limiting the accuracy that can be attrained in certain body wave studies. On the other hand, the data recorded by a

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broad-band station can be employed to determine the crustal receiver structure. Up until now this has generally been done by modelling the receiver function. The receiver function is obtained from teleseismic P-waves using a source equalization procedure (e.g. Phinney 1964; Langston 1979; Owens 1984, 1987; Ammon, Randall & Zandt 1990). The receiver function is dominated by near station P-to-S converted phases and is therefore most sensitive to the shear velocity structure at the site. In this paper we take an alternative approach, as we want to estimate the most pronounced features of the receiver structure. We determine the crustal receiver response from the P-wave coda of a teleseismic event by deconvolution with its source wavelet. This wavelet is assumed to represent the signal at the base of the crust convolved with the instrument response. The crustal response is then inverted to model the structure beneath the station.

Because this approach is akin to the receiver function

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method, it may be elucidating to indicate the similarities and differences between the two techniques, although this paper is not intended to compare the two. In essence, both methods are similar; the crustal structure beneath a station is determined by modelling receiver effects inferred from the teleseismic P-wave coda. The way in which the crustal receiver effects are isolated is, however, different. Instrument response, source and path effects are eliminated in the receiver function method by taking the spectral ratio of the radial to the vertical component. This is an efficient and reliable way of removing the direct P-wave signal and related compressional phases with a similar phase velocity. One pays for this at the cost of the crustal information present on the vertical component. In our approach, the receiver response is estimated by deconvolution with a source wavelet, i.e. the wavelet that is assumed to represent the signal at the base of the crust convolved with the instrument response. As this wavelet is unknown, it must be estimated or guessed. With this approach, care must be taken not to include coda energy into the source wavelet estimate, as this may mask important crustal response effects near the surface, or give rise to unwanted features when fitting later arrivals. The important advantage is that the receiver effects on vertical and horizontal components can now be used to determine both the crustal P- and S-velocity structure.

In this study, we illustrate the method for broad-band receiver responses which are modelled by a plane layered structure. A key element of our approach is that we use a non-linear optimization technique to find the optimum model for this non-linear inversion problem. We employed the technique of conjugate search directions (Nolet 1987), which was successfully applied to relatively low frequency (0.01-0.5 Hz) waveform inversion problems (Nolet, van Tier & Huisman 1986; Nolet 1990; Snieder 1988). The implementation of the technique to this relatively high frequency (0.1-0.5 Hz) problem was set up by Tichelaar (1986), who showed the feasibility of the approach.

In the following, we describe the complete procedure to infer the crustal structure beneath a station from teleseismic data. We examine the possibilities and limitations of the method, and illustrate the procedure for one of the broad-band stations of the Network of Autonmously Recording Stations (NARS) (NE05: Utrecht, The Netherlands). Data and results for other NARS stations are presented in an accompanying paper (Visser & Paulssen 1993).

THE CRUSTAL RESPONSE

The crustal response of a station is defined as the signal generated in the crust beneath the station due to a plane P-wave incident from the mantle. It contains all crustal reverberations and (multiple) conversions which arrive in the coda of the direct P-wave. We estimate the crustal response from the P-wave coda of teleseismic events by deconvolving the first part of the seismogram by an empirically determined source wavelet. This wavelet s(t) is defined to include all effects from the source, mantle propagation, and the instrument response of the station. In this study, for each event, we estimated the source wavelet as the P-wave signal common on all vertical component

NARS recordings. The wavelet is isolated from one of the vertical component NARS recordings, and is sometimes taken from the seismogram of the station under investigation.

After applying a timeshift corresponding to the traveltime of the P phase, the seismic signal starts at t = 0 s, and the first part of the three-component seismogram $\mathbf{u}(t, c)$ can be considered as the convolution of the wavelet s(t) with the crustal response $\mathbf{r}(t, c)$:

$$\mathbf{u}(t, c) = s(t)^* \mathbf{r}(t, c) \tag{1}$$

where c represents the (horizontal) phase velocity of the incident *P*-wave. An approximation of the spectrum of the crustal response can be made by using the deconvolution technique of Langston (1979):

$$\mathbf{R}(\omega, c) = G(\omega) \frac{\mathbf{U}(\omega, c)S^*(\omega)}{\phi_{SS}(\omega)}$$
(2)

where $S^*(\omega)$ denotes the complex conjugate of S, and $\phi_{SS}(\omega) = \max[S(\omega)S^*(\omega), \varepsilon]$ is the autocorrelation of the source wavelet with spectral troughs filled to a fraction c_{SS} of the maximum of the autocorrelation: $\varepsilon = c_{SS} \max[S(\omega)S^*(\omega)]$. $G(\omega)$ is a Gaussian filter that serves to suppress the high frequencies, producing a smooth Gaussian pulse in the time domain:

$$G(\omega) = e^{-\omega^2/4\alpha^2}.$$
(3)

A careful data selection is required for a proper determination of the crustal response. First of all, the seismogram must have a high signal-to-noise ratio to be able to model the relatively weak crustal phases. Secondly, no primary phases should arrive within the time window of interest. This limits the data set to seismograms of deep. teleseismic events. If we assume that most prominent crustal phases arrive with 15s after the direct P-wave, then the events must have a focal depth of at least 50 km for the surface reflected pP and sP phases to arrive later. Furthermore, the epicentral distance must be at least 40° in order to avoid interference with upper mantle triplications and branches of the PP phase. At distances larger than 72° the PcP phase arrives in the time window of interest, but this phase has negligible amplitude at distances beyond 76° (for a surface source). Another constraint is that the source time function must be short and simple. If the source wavelet is too long, then part of the crustal response may be included, and crustal information will be eliminated in the deconvolution procedure. The last criterion of the data selection stems from the modelling technique used in the inversion. As we will make the assumption of a laterally homogeneous crustal structure to compute the synthetic seismograms, the inversion procedure will only yield reliable results when this condition is fulfilled. In practice, the assumption of lateral homogeneity is checked to some extent by inspecting the signal on the transverse component of the seismogram. If the coda wave signal on the transverse component is much higher than the pre-event horizontal noise level, the data must be discarded.

A serious source of error may arise from an incorrect estimation of the source wavelet. It is therefore desirable to improve the signal-to-noise ratio of the receiver response by stacking the responses from a number of events for the same station. This has the additional advantage that an estimate of the variance of the data may be obtained. Stacking can only be applied for clusters of events for which the crustal responses are expected to be similar. This limits the slowness and backazimuth range of the data that can be used for each stack. By synthetic modelling it was verified that, for teleseismic events, slowness variations of 0.008 s km^{-1} do not produce observably different crustal responses. This corresponds to phase velocity variations of the incident *P*-wave of about 1.5 km s^{-1} . By restricting the range of backazimuths per stack to be less than about 10° it is ascertained that lateral variations in the crustal structure around the station will not affect the crustal responses.

In the stacking procedure used in this study, the vertical and radial components of the crustal responses are normalized on the maximum amplitude of the first arrival (per trace), and are given a weight w that is inversely proportional to the noise variance (prior to the *P*-arrival). The vertical (i = 1) and radial (i = 2) components of the *j*th stacked crustal response d_i are given by

$$d^{i}(t, c_{j}, \gamma_{j}) \equiv d^{i}_{j}(t) = \frac{\sum_{k=1}^{N} w^{i}_{k} \cdot r^{i}_{k}(t)}{\sum_{k=1}^{N} w^{i}_{k}}$$
(4)



505

505

Time [s]

Time [s]

510

500

500

norm, ampl.

where N represents the number of crustal responses $r^i(t, c_k, \gamma_k) = r^i_k(t)$ contributing to the *j*th stack, and c_j and γ_j denote the average phase velocity and backazimuth of the *j*th stack, respectively. The (non-normalized) amplitude ratio of the radial to vertical component is remembered for later use in the inversion. An estimate of the variance of a stacked crustal response can be determined from its residue σ^2 .

$$\sigma_j^{2i}(t) = \frac{\sum_{k=1}^{N} |w_k^i \cdot [r_k^i(t) - d_j^i(t)]|^2}{\sum_{k=1}^{N} w_k^{i2}}.$$
(5)

The crustal responses of the NARS stations were determined by the deconvolution procedure with the parameters of the Gaussian filter set to $a = 4\pi$ rad/s (2 Hz) and $c_{SS} = 0.001$. These values were chosen to obtain a stable deconvolution. An example of the deconvolution is shown in Fig. 1, and Fig. 2 illustrates the stacking procedure for two crustal responses of station NE05. The bottom figures show the stacked data on the vertical and radial component, with the residues σ^2 depicted in the panels above.



Figure 1. Deconvolution procedure. (A) Source wavelet of event 860426 determined from the vertical component recording of station NE17 (Toledo, Spain; see Table 1 for event parameters). The time window T1-T2 indicates the source wavelet. (B) Vertical, radial and transverse component of event 860426 recorded by station NE05. (C) Vertical and radial component of event 860426 of station NE05. The vertical bars indicate the time interval corresponding to the source wavelet. (D) Vertical and radial component of the deconvolved seismogram.



Figure 2. Stacking procedure. (a) Vertical and radial component of the deconvolved seismogram of event 860426 recorded by station NE05. (b) Same for event 871003. (c) Vertical and radial component of the stacked crustal response (lower panels) and their residues (upper panels). The time marker T3 indicates the time for which the seismograms are line up.

THE NON-LINEAR INVERSION AND THE INVERSION STRATEGY

Having estimated the (stacked) crustal responses d_j^i from the seismograms, the next step is to determine the crustal model that fits the data best. We therefore need a method to compute synthetic seismograms, a definition of a measure of misfit between data and synthetics, and a procedure that efficiently finds the model with the smallest misfit.

Important constraints to the technique employed for the forward problem of the inversion are that the method should accurately model all features in the data and at the same time be computationally feasible and efficient. We assume that the crustal structure beneath the station can be described by a plane layered medium overlying an upper mantle half-space with the layers specified by their *P*-velocity α , *S*-velocity β , density ρ , and layer thickness *z*. The complete (*P*-*SV*) response of such a model for a *P*-wave incident from below is conveniently calculated by the propagator matrix method (Haskell 1960, 1962; see e.g. Aki & Richards 1980). Convolution of the impulse response of the model with a Gaussian wavelet derived from the vertical component of the crustal response gives the

synthetic response that was used to model the data. The Gaussian wavelet is formed by mirror imaging the first half of the dominant (Gaussian) pulse and its first sidelobe. The wavelet thus formed guarantees a good fit to the data and is contaminated as little as possible by interference with later phases.

The measure of misfit, or norm, F is defined following Nolet (1987):

$$F(\mathbf{m}) = 1/2 \sum_{i=1}^{2} \sum_{j=1}^{N'} W_j \int_0^{T_j} |s_j^i(\mathbf{m}, t) - d_j^i(t)|^2 dt$$
(6)

where \mathbf{m} is the vector of model perturbations relative to a starting model,

$$\mathbf{m} = [(\Delta \alpha / \alpha_0)_1, \dots, (\Delta \alpha / \alpha_0), (\Delta \beta / \beta_0)_1, \dots, (\Delta \beta / \beta_0)_M, (\Delta \rho / \rho_0)_1, \dots, (\Delta \rho / \rho_0)_M, (\Delta z / z_0)_1, \dots, (\Delta z / z_0)_M]^{\mathrm{T}}$$

with M the number of layers in the model. $s_j^i(\mathbf{m}, t)$ is the synthetic seismogram with the vertical component (i = 1) normalized on its maximum amplitude, and the radial component (i = 2) is scaled by the corresponding factor multiplied by the ratio of the vertical to the radial

component measured from the data. The norm is therefore calculated for the data weighted in such a way that the maximum amplitude on both components is equal to one, but the relative amplitude information is maintained. $d_j^i(t)$ denotes the two components of the data for phase velocity c_j . N' represents the total number of crustal responses (stacks) used in the inversion, and T_j is the time span of each crustal response. W_j allows different weighting of the responses. The time span over which is integrated was generally 15 s, corresponding to the length of crustal response obtained from the data.

The inverse problem is non-linear, since, in general, the norm F is an oscillatory function of the model perturbation m_i . We therefore used a non-linear optimization scheme to search in an iterative way for the model that gives an optimum fit to the data, i.e. to minimize the norm F. Following Nolet (1987) we used the method of conjugate directions for the iterative search. To start the inversion, the first search direction is simply taken as the direction of steepest descent:

$$\mathbf{d}^0 = -\mathbf{g}(\mathbf{m}_0) \tag{7}$$

where $g(\mathbf{m}_0)$ is the gradient vector of the norm F at the starting model \mathbf{m}_0 . In the first and subsequent iterations, the local gradient $g(\mathbf{m})$ is approximated by finite differences. $F(\mathbf{m})$ is calculated along the initial search direction until a minimum is passed, and the location of the minimum in this direction is estimated by three-point interpolation. The model thus found is taken as the new starting model for the next iteration. Subsequent conjugate search directions are calculated by the recursive scheme,

$$\mathbf{d}^{i+1} = -\mathbf{g}^{i+1} + \beta_i \mathbf{d}^i \tag{8}$$

where

$$\beta_i = \frac{|\mathbf{g}^{i+1}|^2}{|\mathbf{g}^i|^2} \tag{9}$$

and the procedure is repeated for each new search direction. The search for the optimum model is continued until the norm does not improve by more than 0.5 per cent.

The search path through model space very much depends on the behaviour of the norm, and it is possible that the inversion converges to a local minimum instead of the global minimum. A dedicated inversion strategy is therefore needed to obtain fast and accurate convergence. Before we describe our inversion procedure, it is helpful to gain some insight of the dependence of the norm on the different model parameters.

We have synthesized data for a simple one-layer crust on top of a mantle half space, and computed the norm for a suite of one-layer models. The data are calculated for a model with a crustal *P*-velocity α_c of 6.0 km s^{-1} , a shear velocity β_c of 3.4 km s^{-1} , and Moho depth z_c of 30 km, for phase velocities of 14, 19 and 23 km s^{-1} of the incident *P*-wave. The data are shown in Fig. 3. Three phases can be recognized after the arrival of the direct *P*-wave (at 2 s): the first Moho *P*-wave multiple arriving at about 11 s, the *P*-to-*S* converted phase from the Moho at approximately 6 s, and the *P*-to-*S* converted phase that subsequently reflected as an *S*-wave at the Moho, just visible at the end of the 15-s time window. The norm is computed for a range of models with



Figure 3. Synthetic seismograms for a one-layer crust ($\alpha_c = 6.0 \text{ km/s}$, $\beta_c = 3.4 \text{ km/s}$, $z_c = 30 \text{ km}$) over a mantle half-space ($\alpha_m = 8.1 \text{ km/s}$, $\beta_m = 4.3 \text{ km/s}$). Phase velocity (c) and ratio of radial to vertical component (r/z) are indicated in the upper right corner of each of the vertical components.

the model parameters varying within ± 17 per cent around the optimum values α_c , β_c and z_c .

Fig. 4 shows the norm as a function of *P*-velocity and interface depth for each of the three seismograms separately. The shear velocity was kept fixed to β_c . The norm has an elongated shape with the line $z/\alpha = 5$ s approximately delineating the main valley of the norm. This trade-off between α and z is easily understood by the dependence of the norm on the timing of the *P*-wave multiple. The time difference between the direct *P*-wave and its multiple PpP_mp (nomenclature after Båth & Steffánsson 1966) is given by twice the vertical crustal travel time,

$$t_{PpP_mp-P} = \frac{2z}{\alpha} \sqrt{1 - (\alpha/c)^2}.$$
 (10)

To first order, traveltime variations due to relative perturbations from the optimum model are given by,

$$\Delta t_{PpP_mp-P} = \frac{2z_c}{\alpha_c} \sqrt{1 - (\alpha_c/c)^2} \frac{\Delta z}{z_c} - \frac{2z_c}{\alpha_c} \sqrt{1 - (\alpha_c/c)^2} \frac{\Delta \alpha}{\alpha_c} - \frac{2z_c \alpha_c}{c^2} \frac{1}{\sqrt{1 - (\alpha_c/c)^2}} \frac{\Delta \alpha}{\alpha_c}.$$
 (11)

Substituting the values adopted for our model, it is clear that the dominant contributions to the traveltime variations come from the first two terms in equation (11) with the third term becoming increasingly important as the phase velocity decreases. For high phase velocity, zero time deviations





Figure 4. Norm as a function of *P*-velocity and depth of one-layer crustal models for the seismograms of Fig. 3: (a) for a phase velocity of 14 km/s, (b) 19 km/s, and (c) 23 km/s. The optimum model is marked by a square.

 Δt_{PpP_mp-P} are obtained for models with equal relative perturbations $\Delta \alpha / \alpha_c$ and $\Delta z / z_c$, which causes the $z/\alpha = 5$ s trade-off $(\Delta z / \Delta \alpha = z_c / \alpha_c)$. Different trade-offs between α and z are obtained for smaller phase velocities: the third term of equation (11) amounts to 22 per cent of the second term for c = 14 km s⁻¹.

Another striking feature of Fig. 4 is that the gradient of the norm steepens for larger phase velocity. This is due to the fact that the amplitude of the P-wave multiple increases for larger phase velocity, thus having a stronger influence on the misfit. Note, however, that the P-velocity resolution is

almost equal for the three seismograms. This illustrates the relatively weak dependence of the norm (or the PpP_mp amplitude) on the *P*-velocity contrast at the Moho interface.

For a better insight into the dependence of the misfit on all three model parameters, we show in Figs. 5a and b the norm as a function of β and z, and α and β , respectively, for the data with a phase velocity of 14 km s⁻¹. The third model parameter was kept fixed to its true value. Again, there are clear trade-offs that can be related to the timing of the various arrivals. For the *P*-to-*S* converted phase (*Ps*), the time difference with the direct *P*-wave is given by

$$t_{P_{S}-P} = \frac{z}{\beta} \sqrt{1 - (\beta/c)^2} - \frac{z}{\alpha} \sqrt{1 - (\alpha/c)^2}.$$
 (12)

Traveltime perturbations due to relative variations in α , β , or z are dominated by the c-independent contributions,

$$\Delta t_{P_{S}-P} = z_c \left[\frac{1}{\beta_c} - \frac{1}{\alpha_c} \right] \frac{\Delta z}{z_c} + \frac{z_c}{\alpha_c} \frac{\Delta \alpha}{\alpha_c} - \frac{z_c}{\beta_c} \frac{\Delta \beta}{\beta_c} \,. \tag{13}$$

Substituting the values of our model in equation (13) it is obvious that the trade-offs between β and z, and α and β , seen in Figs 5a and b, can be explained by zero time deviations of the *Ps* phase. Similarly, the secondary (negative) trade-off between α and z presented in Fig. 4a stems from the timing of the *Ps* phase. For $c = 14 \text{ km s}^{-1}$, the c-dependent terms omitted in equation (13) amount to 5 per cent, 9 per cent, and 3 per cent of the contributions of $\Delta z/z$, $\Delta \alpha/\alpha$, and $\Delta \beta/\beta$ of equation (13), respectively; they



Figure 5. (a) Norm as a function of S-velocity and depth for the first seismogram of Fig. 3, and (b) as a function of P-velocity and depth.

decrease rapidly for larger phase velocity. The weak dependence of the timing of the converted phase on the phase velocity implies that the trade-off between the different parameters is hardly reduced by using more seismograms with different phase velocity. The same conclusion was reached by Ammon *et al.* (1990) in a study of synthetics tests of receiver function inversions.

The third phase contributing to the norm, although only partly present in this 15 s time window, is the *PS*-converted and Moho-reflected PsS_ms phase. The relative timing of this phase is given by

$$t_{PsS_{m}s-P} = \frac{3z}{\beta} \sqrt{1 - (\beta/c)^2} - \frac{z}{\alpha} \sqrt{1 - (\alpha/c)^2}$$
(14)

and, to good approximation, its traveltime perturbations by

$$\Delta t_{PsS_{m}s-P} = z_c \left[\frac{3}{\beta_c} - \frac{1}{\alpha_c} \right] \frac{\Delta z}{z_c} + \frac{z_c}{\alpha_c} \frac{\Delta \alpha}{\alpha_c} - \frac{3z_c}{\beta_c} \frac{\Delta \beta}{\beta_c} \,. \tag{15}$$

Variations in z and β have a stronger effect on the timing of this phase than on the earlier two phases. The influence on the norm in our example is nevertheless limited because the phase partly falls outside the time window.

In summary, the timing and amplitude of a phase both affect the norm, but in a different way. The resolution of the model parameters depends most strongly on their influence on the timing of the phases and may cause strong trade-offs. The amplitudes of the phases determine the amplitude variation of the misfit associated with perturbations in the model parameters. The phase velocity has a stronger effect on the timing of the *P*-wave multiples than of the converted phases. In our example, the elongated shape of the contours in Fig. 5a and b show that the *P*-velocity and depth are better constrained than the *S*-velocity, with, from Fig. 4a, the *P*-velocity and depth almost equally well resolved.

Theoretically, the trade-off between *P*-velocity and layer thickness may be reduced by using several crustal responses with different phase velocities in the inversion. In practice, however, the effect is limited. Fig. 6a shows the norm for the three input crustal responses together. The norm is divided by three for an easy comparison with the single-seismogram results of Fig. 4. Note that the resolution in *P*-velocity and depth is hardly better in case of the three seismograms compared to the single-seismogram case. The main advantage of the larger number of seismograms is that the misfit is smoothed and the local minima are reduced. This effect is important, particularly for the inversion of more complicated structures, when the chance of convergence to local minima is larger.

Smoothing of the norm can be advantageous to achieve fast convergence to the global minimum, because the effect of small local minima is reduced. Another way to smooth the norm is to smooth the data and synthetics by frequency filtering. We have applied low-pass \cos^2 -filters to the time series to reduce the oscillatory behaviour of the norm. Fig. 6b shows the effect of filtering by a \cos^2 -filter with a cut-off frequency of 0.2 Hz. The inversion can be run several times with gradually increasing cut-off frequency, taking for each new inversion the result of the previous inversion as starting model. Such a procedure stabilizes the inversion process, reducing the chance of convergence to a local minimum.

An efficient way to optimize the inversion procedure is to





Figure 6. (a) Norm as a function of *P*-velocity and depth for the three seismograms of Fig. 3. The values are divided by three for easy comparison with Fig. 4. (b) Norm after filtering data and synthetics (phase velocity 14 km/s) by a low-pass \cos^2 -filter.

restrict the number of free parameters. Theoretically, the model space has a dimension of 4M, where the variables α , β , ρ and z for each of the M layers represent the model parameters. Parameters that are badly constrained by the data, such as the density, can be eliminated by multiple gridding, i.e. by projecting the model space onto a subspace of smaller dimension. In practice, this means that parameters are kept fixed during the inversion, or coupled to other parameters by known relationships. For example the density may be kept fixed to an a priori value or coupled to the P-velocity. The S-velocity can be coupled to the *P*-velocity by the relation $\alpha = 1.73\beta$ under the assumption of a Poisson medium. In general, this latter relation will be a good first approximation, although it may not be accurate at all depths. Therefore, once the inversion has reached convergence in the smaller model subspace, the coupling between α and β should be released. An important aspect of multiple gridding is that the computations are speeded up significantly in a smaller subspace. It is an efficient way to achieve fast convergence, even if in subsequent inversions the preconditioning is relaxed.

Our inversion strategy is based on the above inferences. The density was coupled to the *P*-velocity by the ratio given in the starting model. The parameters of the upper mantle half-space were kept fixed to $\alpha = 8.1 \text{ km s}^{-1}$, $\beta = 4.30 \text{ km s}^{-1}$, and $\rho = 3.30 \text{ g cm}^{-3}$. The first inversion was

generally performed with a three- or four-layer starting model based on *a priori* information. The *P*- and *S*-velocity were coupled in the first inversion run, and a \cos^2 -filter was used with a cut-off frequency of 0.2 Hz. The cut-off frequency of the filter was slowly increased in subsequent inversions until it reached 1.6 Hz, the highest frequency present in the stacked data. In the following inversion, the coupling between the *P*- and *S*-velocity was removed. This inversion process was repeated, each time increasing the number of layers by one. When the adding of extra layers did not improve the fit, the inversion was considered to be finished.

This strategy proved to be a reliable method to obtain the crustal structure under the NARS stations. A check on the stability of the result was made by re-running the inversion for different starting models. In general, these inversions resulted in models whose parameters were in agreement within 5 per cent.

APPLICATION: THE INVERSION FOR STATION NE05

Crustal *P*- and *S*-velocity functions have been determined for 9 of the 18 locations where NARS stations were installed during the period from 1983 to 1987 (Visser & Paulssen 1992). In this paper we describe the inversion process to demonstrate the possibilities and limitations of the technique, and illustrate how features in the data relate to structural features of the model.

The inversion procedure is shown for the crustal structure of station NE05 (Utrecht, The Netherlands; 52.088N, 5.172E). Fig. 7 shows the data that were used in the inversion. The stacked crustal responses (lower panels, solid lines) and their residues (upper panels) are shown for the vertical and radial component. The crustal responses are obtained for northeasterly to easterly backazimuths with average phase velocities of 14.7, 19.3 and 23.6 km s^{-1} . Event parameters of the seismograms contributing to the stacks are given in Table 1. Note that the seismograms which contribute to response 2 might contain a phase reflected at the top of the D" layers (Weber & Davis 1990). One of the seismograms (event 830226) indeed shows evidence for such a phase, whereas a weak arrival might be present on another (event 840423). However, because of the different ray paths we do not expect such phases to be coherent. The vertical component of the stacked response does now show evidence for signal generated in the D"



Figure 7. Observed (solid lines) and best fitting synthetic (dashed lines) crustal responses for station NE05. Data are obtained from the events listed in Table 1. Phase velocity, (back)azimuth, and ratio of radial to vertical component are given in the upper right corner of the vertical component. The synthetic responses are calculated for the model of Fig. 8.

Table 1. Event parameters.

response	event code	Δ (°)	backaz. (°)	depth (km)	phase vel. (km/s)	average phase vel.
1	860426	48.0	817	187	14 47	14.7
	871003	48.3	81.5	80	15.00	•,
2	830226	76.0	19.5	57	19.15	19.3
	870518	74.2	24.5	554	19.35	
	840423	75.5	25.8	415	19.45	
	840420	73.7	23.5	518	19.52	
3	840424	87.9	38.9	424	23.62	23.6

layer. Because the third response is obtained for one seismogram only, it was given a weight W = 0.5 instead of W = 1.0 in the inversion. The synthetics computed for the final model of the inversion are also shown in Fig. 7 (dashed lines).

One of the striking features of the response of NE05, is that the first arrival on the radial component of the seismogram appears to be delayed with respect to the direct P-arrival on the vertical component (see also Fig. 1). Such an apparent delay is also recognized for other stations of the NARS array (Visser & Paulssen 1993), and a similar phenomenon was observed by Owens & Crosson (1988) on broad-band data of a local network in Washington state. Another notable feature is the high amplitude level on the radial component (roughly half of that on the vertical component) which only slowly decreases with time. The interpretation of these features is described in the following, where we also give an outline of the inversion procedure.

First, we focused on the 'delay' on the radial component, because it has a large contribution to the norm. As it must be caused by the upper crustal structure, we selected a number of four-layer starting models with widely varying upper crustal structure. The layers represent: (1) a layer of unconsolidated sediments, (2) a layer of consolidated sediments, and (3) an upper and (4) a lower crustal velocity typical for Caledonian areas in northwestern Europe (Meissner 1986). None of the models with the P- and S-velocities coupled according to a Poisson medium resulted in a significant decrease of the norm during the inversion. It appeared that for starting models with a Poisson's ratio of about 0.4 in the uppermost layer stable convergence was achieved, with the synthetics modelling both the 1.3-s delay of the first arrival on the radial component and the high amplitude coda on this component (compare for instance with Fig. 3). The first distinct arrival on the radial component is interpreted by the inversion as a P-to-S converted phase, and the subsequent high-amplitude signal is caused by a sequence of SV-multiples from the first interface. The very low values for P- and S-velocity in the uppermost layer are consistent with the inferred nearvertical incidence of the direct P-wave, thereby explaining its apparent absence on the radial component. It also explains the weak correlation between the two components of the responses. A low S-velocity in the uppermost layer yields a large S-velocity increase across the first interface which is required to produce the high-amplitude SVmultiples.

The conversion depth of the first clear (*P*-to-*S* converted) arrival on the radial component is inferred to be approximately 11 km; this depth is well constrained by the



Figure 8. Final model of the inversion for station NE05.

timing of the phase, 1.3 s after the direct *P*-wave. The absolute value of *S*-velocity below this interface is not particularly well resolved, although a large velocity contrast is required at this depth. A small velocity discontinuity was determined by the inversion at a depth of approximately 5.5 km. This finer detail in the model gave a small but consistent improvement to the norm.

In subsequent inversion runs extra layers were added to assess the sensitivity of the norm to structure at greater depths. It appeared that the addition of detailed layering below the third interface did not greatly enhance the fit. This is due to the fact that the response of the upper part of the crust dominates the signal generated in the lower crust. This was also evident from the inversion results, as different inversions converged to models with varying lower crustal structure which gave a nearly equally good fit.

The optimum model of the inversions is presented in Fig. 8. The misfit of data and synthetics is smaller than the error estimated from the variance of the data. This indicates that our optimum model is one of a range of models that give an acceptable fit to the data. All velocity structures that give an acceptable fit to the data may be determined by a systematic search through model space (Ammon *et al.* 1990). Alternatively, a Monte Carlo experiment will give the *a posteriori* density distribution from the *a priori* density distribution estimated from the residues (e.g. Cary & Chapman 1988). If the misfit behaves sufficiently smoothly in the region around the optimum model, it is possible to estimate the uncertainty of the parameters by a local Taylor expansion around the optimum model (Nolet 1990). Any of these resolution investigations would be a study in itself, and we have not performed a detailed resolution analysis. A rather qualitative estimate of the resolution is obtained by perturbing each of the model parameters by roughly 10 per cent (keeping the other parameters fixed) and looking at the effect on the norm. It appears that the upper crustal structure of the model for station NE05 is relatively well constrained, but its lower crustal structure is poorly resolved by the data and may not be reliable. The shear-velocity and depth of the first three layers are best resolved, with 10 per cent perturbations giving in general unacceptable large increases to the norm. Although the upper crustal P-velocity gives the optimum fit to the data, the P-velocity is not very well constrained. From the inversion results it was already clear that the lower crustal structure is not very well resolved, as inversions with different starting models converged to models with different lower crustal structure. Unfortunately, there are no results from refraction or reflection experiments available for the area. Our model can therefore not be compared with results from other independent studies. We may remark that the upper crustal P-velocities are rather low. The 27-km Moho-depth agrees reasonably with estimates by Remmelts & Duin (1990) and Meissner, Wever & Flüh (1987) of 30 km, but this fair agreement may be coincidental as the lower crustal structure is poorly resolved.

DISCUSSION AND CONCLUSION

The basic idea of the method described in this paper is analogous to the more classical spectral ratio method, or receiver function: to determine the crustal structure from the *P*-wave coda. The main advantage of our method is that the radial and vertical component are used to model the crustal *P*- and *S*-velocity structure. The disadvantages arise from the fact that the source wavelet, the wavelet incident at the base of the crust, must be estimated from the data. Thus, only events with short source time functions can be employed. A wrong estimation of the source wavelet may lead to an incorrect estimate of the crustal response, which may give rise to unwarranted features in the model. Stacking of the crustal responses is therefore essential to reduce the influence of such errors.

For layers where both the P- and the S-velocity are well resolved it is possible to estimate Poisson's ratio of the layer. The partitioning of energy between the radial and vertical component is an important piece of information which constrains the P- and S-velocity in the uppermost layer. Thus, the top layer under NARS station NE05 could confidently be interpreted as a layer on unconsolidated sediments from its high Poisson's ratio. Although it is also possible to use radial-to-vertical amplitude ratio in the receiver function (Ammon 1991), it has not been done in many former studies.

Our synthetic test of a one-layer crust showed that the P-velocity and interface depth were better resolved than the S-velocity by the combined modelling of multiples and converted phases. The timing of the P-wave multiples appears to be more sensitive to variations in phase velocity than the timing of the converted phases. Although in theory the resolution of P-wave velocity will increase by using

seismograms with different phase velocity, the effect is limited in practice. Yet, an important advantage is that a larger number of receiver responses helps to smooth the misfit function, and the effect of errors in the data is further reduced.

The non-linear inversion via conjugate search directions appeared to be a fast and robust technique, although it requires 'adaptive preconditioning' to avoid convergence to a local minimum of the norm. The stepwise inversion strategy, in which the model parameters become less and less constrained in subsequent inversion runs, proved to be a convenient way to converge to the optimum model.

The application of the method to data of station NE05 showed that distinct features of the model, the layer of unconsolidated sediments and the velocity gradient at 11 km depth, are relatively well constrained. Other features of the velocity structure at this station have a smaller influence on the crustal response, and are therefore less well resolved.

It is clear that the method is mainly sensitive to large velocity contrasts which produce high-amplitude reflected and converted phases. These phases give a large contribution to the norm or misfit, and the features in the crustal velocity structure that generate them are therefore best resolved. The method is an efficient technique to investigate the crustal seismic velocity structure using available seismological data. By treating the vertical and horizontal components of the seismogram separately, optimum use is made of the observations. Even where the seismological data do not constrain all of the detail in crustal structure, the crustal response itself is accurately determined, and can be used to correct new data.

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