Antwoorden tentamen 5 november 2009

Deel 1

1. (a) $x = 1, \;\; y = 2/3, \;\; z = -2/3$
   (b) $x = -3 + 2y, \;\; z = -7 + 4y$ ofwel
   $$\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ -7 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} y = \begin{pmatrix} -3 \\ 0 \\ -7 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} t$$
   (c) a: punt b: lijn

2. $a = 1$

3. (a) $-x + 4y + z = 5$
   (b)
   $$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} t$$
   of
   $$\frac{x-2}{-1} = \frac{y-1}{4} = \frac{z+1}{1}$$
   (c) Afstand $d = 1$.

4. (a)
   $$A^{-1} = \frac{1}{2k} \begin{pmatrix} 1 & -3k+1 & 2k-1 \\ 2 & 2 & -2 \\ -1 & k-1 & 1 \end{pmatrix}$$
   Singulier als $k = 0$.
   (b)
   $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$
   Als oplossing niet met A^{-1} berekend is, wordt oplossing fout gerekend.

5. $$(B^T AB)^T = B^T AB$$

Deel 2

1. (a)

$$\lambda_1 = 0:$$

$$\mathbf{r}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\lambda_{2,3} = 1:$$

$$\mathbf{r}_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{r}_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

   (b)

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} \frac{1}{2}\sqrt{2} & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{1}{2}\sqrt{2} & 0 & 0 \end{pmatrix}$$

   $MC = CD$

2. (a) Cilindrisch: $r = 2,\ \theta = 5/4\pi,\ z = -2$
   Sferisch: $r = 2\sqrt{2},\ \theta = 3/4\pi,\ \phi = 5/4\pi$
   (b) πa^2

3. Volume $= 4.5$

4. (a)

$$\nabla\phi = \begin{pmatrix} -3y \\ -3x \\ 2z \end{pmatrix}$$

$$\nabla^2\phi = 2$$

   Richtingsafgeleide = $-\sqrt{3}$

   (b)

$$\nabla \cdot \mathbf{V} = 5xy$$

$$\nabla \times \mathbf{V} = \begin{pmatrix} xz \\ -yz \\ y^2 - x^2 \end{pmatrix}$$

   $\mathbf{V}$ is niet conservatief.

5. (a) Gauss:

$$\iiint_{\text{volume of } \tau} \nabla \cdot \mathbf{V}\, d\tau = \iint_{\text{surface of } \tau} \mathbf{V} \cdot \mathbf{n}\, d\sigma$$

   Stokes:

$$\oint_{\text{curve bounding } \sigma} \mathbf{V} \cdot d\mathbf{r} = \iint_{\text{surface } \sigma} (\nabla \times \mathbf{V})\cdot\mathbf{n}\, \mathbf{d}\sigma$$

   (b) 2