Supplementary Material to 'Robust Normal Mode Constraints on Inner Core Anisotropy From Model Space Search' by C. Beghein and J. Trampert.

Data

Two data sets were tested : the first one consisted of older normal mode splitting measurements (*1, 2*) (with 15 degree two and 7 degree four structure coefficients), and the second one consisted of the most recent splitting measurements that followed the great Bolivia and Kuril Islands earthquakes in 1994 (*3–5*) (with 22 degree two and 16 degree four data). The choice of data set one was based on the modes employed by Tromp (*6*) to derive his first model of inner core anisotropy. However, we decided to remove mode ${}_6S_3$, due to a possible overlapping with mode ${}_3S_8$, and ${}_{13}S_2$, since it has been demonstrated (*7*) that different splitting functions could explain the spectrum of mode ${}_{13}S_2$ equally well. We also discarded the measurements made by Giardini, Li & Woodhouse (*8*) because their degree two splitting coefficients were systematically smaller than coefficients determined in other studies (*1–5*). In the end, data set one was composed of degree two measurements for modes ${}_2S_3$, ${}_3S_2$, ${}_8S_5$, ${}_9S_3$, ${}_{11}S_4$, ${}_{11}S_5$, ${}_{13}S_3$, ${}_{15}S_3$, ${}_{16}S_6$, ${}_{18}S_4$, ${}_{20}S_5$, ${}_{21}S_6$, ${}_{23}S_5$, ${}_{25}S_2$ and ${}_{27}S_2$; degree four data were modes ${}_2S_3$, ${}_3S_2$, ${}_8S_5$, ${}_9S_3$, ${}_{11}S_4$, ${}_{11}S_5$ and ${}_{13}S_3$. Data set two was composed of degree two measurements for modes ${}_2S_3$, ${}_3S_2$, ${}_5S_3$, ${}_7S_4$, ${}_7S_5$, ${}_8S_1$, ${}_8S_5$, ${}_9S_3$, ${}_{11}S_4$, ${}_{11}S_5$, ${}_{13}S_1$, ${}_{13}S_3$, ${}_{16}S_5$, ${}_{16}S_7$, ${}_{17}S_1$, ${}_{18}S_3$, ${}_{18}S_4$, ${}_{21}S_6$, ${}_{21}S_7$, ${}_{21}S_8$, ${}_{23}S_5$ and ${}_{27}S_1$, and degree four measurements for modes ${}_2S_3$, ${}_3S_2$, ${}_5S_3$, ${}_7S_4$, ${}_7S_5$, ${}_8S_5$, ${}_9S_3$, ${}_{11}S_4$, ${}_{11}S_5$, ${}_{13}S_3$, ${}_{16}S_5$, ${}_{16}S_7$, ${}_{18}S_3$, ${}_{21}S_7$, ${}_{21}S_8$ and ${}_{23}S_4$. Original error bars for degree four data of mode ${}_2S_3$ were increased in data set two to account for discrepancies between the measurements. Modes ${}_3S_8$, ${}_6S_3$ and ${}_{13}S_2$ were discarded as in data set one together with ${}_3S_1$ which is also difficult to measure.

Inversions: mantle corrections and data fit

In two recent studies (*9, 10*), a model space search technique was employed to find families of mantle models that fit normal mode splitting measurements. These ensembles of mantle models were randomly sampled, according to their associated probability density functions, to correct the data. They gave rise to a family of inner core models, represented by the shaded grey areas in Fig. (1) of the main paper (the limits correspond to two standard deviations). Six mantle models (*11–16*) resulting from the inversion of seismological data were also employed, yielding the models represented in color.

The models of inner core anisotropy resulting from those inversions fit the recent normal mode data set with a χ-misfit between 3.5 and 5.5 (depending on the mantle model and the damping). For comparison, the inner core model obtained by Tromp (*6*) gives $\chi \simeq 5.6$ when data are corrected with SKS12WM13 (*16*). Without inner core anisotropy, the χ-misfit varies between 9.1 and 10.9, depending on the mantle model. Without mantle correction and without inner core anisotropy, $\chi \simeq 12.2$.

The neighbourhood algorithm

The neighbourhood algorithm (NA) (*17*, *18*) consists of two stages. During the first stage, the model space is surveyed to identify the good data-fitting regions. In the second stage, the NA employs a Bayesian approach to compute the posterior covariance matrix and the marginals associated with each model parameter. The two stages require the tuning of two parameters that will insure, firstly, that all the good data-fitting areas are sampled, without being trapped in a local minimum, and, secondly, that the integrals of the second stage converge. In our study, fifteen model parameters were being searched: five spline coefficients for three anisotropic parameters. The NA divides the model space into Voronoi cells, which are uniquely defined and space-filling. Each cell contains one model, and can therefore be associated to a misfit value. The measure of misfit we chose is the usual χ-misfit. During the first stage, n_s models are iteratively generated in the n_r best data-fitting cells, and the search is directed towards the regions where the fit is the best. To keep the search as broad as possible, the two tuning parameters n_s and n_r were kept equal and we took care of the stability of our results by testing values of n_s ranging from 10 to 300. The total number of models generated was increased with the tuning parameters. When sampling the space with 300 best data-fitting cells at each iterations, we generated a total of 150,000 models. During the second stage of the NA, the models generated in the first stage are resampled to produce probability density functions associated with each model parameter, or couple of model parameters, as well as the trade-offs among them. The number and the length of the random walks that are performed for this resampling also require the tuning of two parameters. We achieved the convergence of the integrals with 15 walks of 6,000 steps each. Convergence was equivalently obtained using longer walks (walks of 15,000 steps).

Model space search: mantle corrections and data fit

We used data set two described in the data section (see the above). No covariance was assumed between degree two and degree four data. Data errors were given by the authors of the measurements (*3*–*5*), and were assumed to be Gaussian distributed. The fit to the strongly split mode ${}_3S_2$ is very good, both at degree two and degree four. It is interesting to note that removing ${}_3S_2$ from the data does not change the results for α, but does increase the uncertainties on β and γ at a radius of 300 km. As explained, we discarded ${}_6S_3$ and ${}_{13}S_2$ also highly sensitive to the inner core structure. Tests including ${}_6S_3$ and ${}_{13}S_2$ in the NA produced models that highly degraded the fit to mode ${}_3S_2$ and to travel-time data at high epicentral distances. Including complementary measurements of Widmer (*2*) didn't change our results either. We finally tested that the introduction of zonal degree two and four density perturbations in the parametrization did not alter the results. The correlation between the anisotropic parameters and the fit to the data did not change either. We used different mantle corrections : SB10L18 (*15*), SKS12WM13 (*16*) and the most likely V_p, V_s and density mantle model derived with the NA (*10*). Our results did not strongly depend on the mantle correction. Only for mantle model SKS12WM13 (*16*) our analysis produced a significant proportion of inner core models with a negative P-wave anisotropy at the inner core boundary not supported by $150°$ travel time data. We only show the models obtained using the mantle model of Resosvky and Trampert (*10*). It was not derived from an inversion (the solution is, therefore, not contaminated by any regularization) and, in contrast with most mantle models, no scaling was assumed between density and velocity anomalies.

Travel time predictions

Random deviates were drawn from the marginal posterior probability density functions of each model parameter obtained from the NA to make predictions of travel time anomalies. One hundred thousands models were randomly generated, and predictions of travel time anomalies were computed for all of them, with different ray angles. We thus had a distribution of predictions for each ray angle, and the width of these distributions, corresponding to 95% of the predictions, was used to plot the range of travel time predictions in Fig. 4 of the main paper.

References and Notes

1. M. Ritzwoller, G. Masters, F. Gilbert, *J. Geophys. Res.* **93**, 6,369 (1988).

2. R. Widmer, G. Masters, F. Gilbert, *Geophys. J. Int.* **111**, 559 (1992).

3. J. Tromp, E. Zanzerkia, *Geophys. Res. Lett.* **22**, 2,297 (1995).

4. X. He, J. Tromp, *J. Geophys. Res.* **101**, 20,053 (1996).

5. J. Resovsky, M. Ritzwoller, *J. Geophys. Res.* **103**, 783 (1998).

6. J. Tromp, *Nature* **366**, 678 (1993).

7. J. Durek, B. Romanowicz, *Geophys. J. Int.* **139**, 599 (1999).

8. D. Giardini, X.-D. Li, J. Woodhouse, *J. Geophys. Res.* **93**, 13,716 (1988).

9. C. Beghein, J. Resovsky, J. Trampert, *Geophys. J. Int.* **149**, 646 (2002).

10. J. Resovsky, T. Trampert, *Nature* (2002). Submitted.

11. J. Resovsky, M. Ritzwoller, *J. Geophys. Res.* **104**, 993 (1999).

12. G. Masters, S. Johnson, G. Laske, H. Bolton, *Phil. Trans. R. Soc. London* **354**, 1,385 (1996).

13. J. Ritsema, H.-J. van Heijst, *Science* **83**, 243 (2000).

14. X. Li, B. Romanovwicz, *J. Geophys. Res.* **101**, 22,245 (1996).

15. G. Masters, G. Laske, H. Bolton, A. Dziewonski, *Earth's Deep Interior: Mineral Physics and Tomography From the Atomic to the Global Scale, Seismology and Mineral Physics, Geophys. Monogr. Ser.*, S. Karato, ed. (AGU, Washington, D.C., 2000), vol. 117, pp. 63–87.

16. W.-J. Su, R. Woodward, A. Dziewonski, *J. Geophys. Res.* **99**, 6,945 (1994).

17. M. Sambridge, *Geophys. J. Int.* **138**, 479 (1999).

18. M. Sambridge, *Geophys. J. Int.* **138**, 727 (1999).