Lithosphere Applications of the Finite Element Method (LAFEM)

An introductory course to practical solutions of static continuum mechanics and heat conduction, with a specific emphasis on the finite element code GTECTON, and related software. The course revolves around lab exercises in which participants explore simplified finite element models of typical lithospheric problems.

Background

Participants are expected to have an operational background in

1. algebra, (solving) differential equations, and tensor analysis,
2. classical physics (mechanics and heat) and continuum mechanics.

Experience with a UNIX-like computer will be highly beneficial, but is not a requirement.

Day 1 (Monday May 8, 2017)

Lecture 1: Introduction to Finite Elements

> Mechanical equilibrium equation, finite element approaches. Elasticity equations, going from 3D to 2D.

Lab 1: Flexure of a 2D thin elastic beam

> Introduction: analytical solution to the engineering flexure equation, clamped on one end, force on other end.
>
> Creating and checking a tessellation in a rectangular domain (*triangle*). Creating a GTECTON input file (*gedit*). Applying and verifying boundary conditions (*plnplt* and *GMT*) Computing a finite element solution (*pln*). Plot vertical surface deformation (*plnplt* and *GMT*).

Day 2 (Tuesday May 9, 2017)

Lecture 2: Basics of Finite Elements

> Shape functions and element equations. Shape functions, element equations, stiffness matrix. Nodal boundary conditions.
>
> Quantifying results, convergence and solution quality. Tesselation, boundary conditions, and other issues affecting the numerical solution.

Lab 2:

> Compare Lab 1 results with analytical solution; where did things go wrong? Tesselation exercise.

Day 3 (Wednesday May 10, 2017)

Lecture 3: Solvers and Practical issues and questions

> Direct and iterative solvers (PETSc). Stiffness matrix topology. Discussion of flexure lab problem.

Lab 3: A converged solution to 2D elastic flexure in engineering

Tesselation convergence test for Lab 1 problem.

Day 4 (Thursday May 11, 2017)

Lecture 4: Element boundary conditions

Numerical integration. Boundary tractions and Winkler pressures. Euler angles. Coloring special nodes, element faces and elements. Scalar, vector and tensor quantities. Flexure of an elastic lithosphere.

Lab 4: Elastic lithosphere on an inviscid asthenosphere

New rectangular model domain (*triangle*). Creating a GTECTON input file (*gedit*) Automatic selection elements for applying boundary conditions (*picknps* and *elmside*) Verifying boundary conditions (*plnplt* and *GMT*) Compute a finite element solution (*pln*) Plot displacements and stresses (*plnplt* and *GMT*).

Day 5 (Friday May 12, 2017)

Lecture 5: large deformation models and gradient boundary conditions

Symmetry, anti-symmetry, periodic, and no-tilt BC's. Compatibility and completeness. Force balance and residual force update.

Lab 5: Buckling of a 2D elastic plate

Rectangular domain, subject to in-plane traction and bifurcation-forcing. Apply theoretical limit traction magnitude, with and without residual force update.

Day 6 (Monday May 15, 2017)

Lecture 6: Kinematically driven fault displacement

Faulted (or split) nodes and their FE implementation. Mesh design and auxilliary fault segments. Practical ingredients of the workflow.

Lab 6: Elastic half-space with an imposed fault slip

Reproduce the semi-analytical solution for well-chosen boundary conditions and mesh. Use residual force update.

Day 7 (Tuesday May 16, 2017)

Lecture 7: Dynamically driven fault displacement

Slippery nodes and their FE implementation.

Lab 7: Elastic half-space with an dynamic fault slip

Same as Lab 6, but for dynamic boundary conditions. Again use the residual force update.

Day 8 (Wednesday May 17, 2017)

Lecture 8: Stress relaxation

Plasticity, viscosity and their implementation in GTECION. Time stepping issues. Overview of the release version of the FE package (2011), of the new aspects of the development version of the package (2013), and upcoming changes.

Lab 8: Post-seismic relaxation

Variation of Lab 6, where an elastic layer has a thrust earthquake occuring on it and deeper layers respond subsequently by linear viscous flow.

Day 9 (Thursday May 18, 2017)

Lecture 9:

Conduction equation, FE solution, initial conditions, boundary conditions

Lab 9:

Steady state solution for geotherm in continental lithosphere with variable thickness crust.

Day 10 (Friday May 19, 2017)

Lecture 10:

Modeling philosophy, why model, good models bad models.

Participant presentations:

Research questions, model approach/setup to solving the question.

Lab 10:

Setting up of a first FEM to start solving your research question.