Scaling laws for the equilibrium internal temperature in the stagnant lid regime with depth-dependent rheologies

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Summary

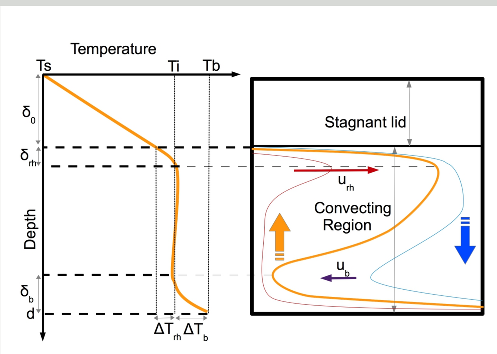
We study the effect of pressure on a convecting system in the stagnant lid regime. We find that the internal temperature is strongly affected by the pressure-dependence of the viscosity even when radioagenic heating is the dominant heat source. We propose a new scaling law of heat flux and internal temperature based on an extended model of the classical boundary layer theory.
The internal temperature is found to strongly depend on the internal lateral and vertical viscosity contrasts. The heat flux is largely affected by the pressure-dependence of the viscosity since it depends on the internal temperature, but its expression remains close to previous models.
In bottom heated domains, a pressure-dependent rheologies tend to make the internal temperature of planets lower. When radiogeneic heating dominates, the internal temperature is larger if the rheology depends on pressure. Simulations (StagYY, spherical annulus geometry) in which radiogenic heating is time-dependent are presented.

Analytical Nusselt number and internal temperature

NO INTERNAL HEATING

- **Requirement at the equilibrium**, the energy dissipation is:

$$\Phi_{diss} = \int_V \tau_{ij} \frac{\partial u_i}{\partial x_j} dV = \frac{\alpha g V}{C_p} F \left(1 - \frac{1}{Nu}\right),$$



where V is the volume of the domain, τ_{ij} the deviatoric stress tensor, u_i the velocity, x_j the space coordinate, α the thermal expansivity, g the gravitational acceleration, C_p the heat capacity at constant pressure, F the heat flux and Nu the Nusselt number.

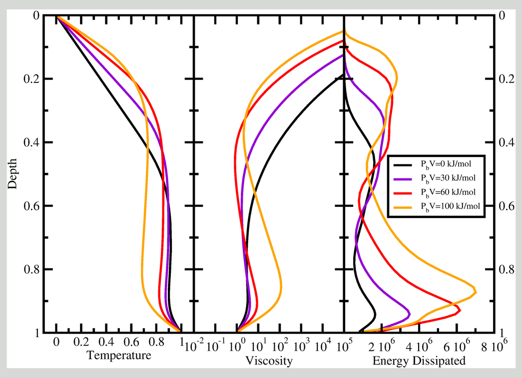
- **Assumption: homogeneous viscous disspation** in the internal region:

$$\eta_{P(z),T_i} \dot{\epsilon}^2_{P(z),T_i} = Cst \qquad \forall z,$$

where T_i is the internal temperature in the convecting domain.
- **Pressure-temperature dependent viscosity**:

$$\eta_A = \eta_0 \exp\left(\frac{E+PV}{nRT}\right)\left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0}\right)^{\frac{1-n}{n}},$$

where η_0 is a reference viscosity, E is an activation energy, V is an activation volume, n is the stress exponent, R is the universal gaz constant, $\dot{\epsilon}$ is the second invariant of the strain rate tensor and $\dot{\epsilon}_0$ is a reference diffusive strain rate $\dot{\epsilon}_0 = \kappa/d^2$ with κ the thermal diffusivity and d the characteristic size of the considered domain.



⇒ **Dimensionless parameters**

- $\theta = \frac{\Delta T (E+P_i V)}{R T_i^2}$, Frank-Kamenetskii parameter,
- $Ra_{rh} = \frac{\alpha \rho g \Delta T d^3}{\kappa \eta_0 \exp\left(\frac{E+P_{rh}V}{nRT_i}\right)}$, Rayleigh Number,

⇒ **Internal temperature**

$$T_i = T_b - \Delta T \theta^\beta Ra_{rh}^\gamma \left(\Delta\eta_v^{\frac{n}{2n+2}} + \lambda \Delta\eta_l^\Gamma\right).$$

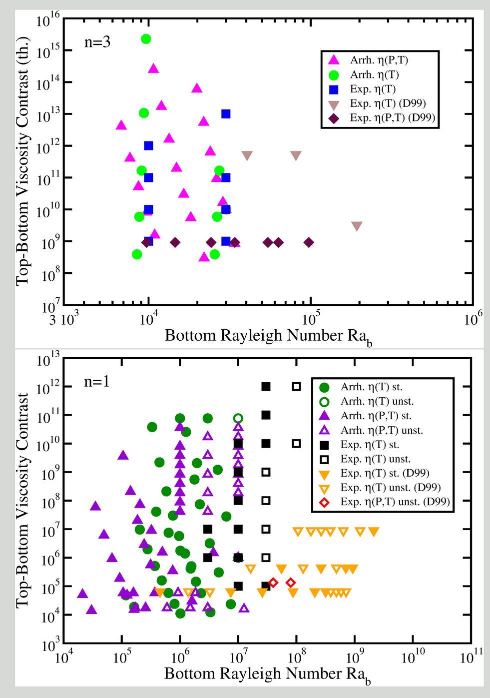
where $\Delta\eta_l$ and $\Delta\eta_v$ are lateral and vertical viscosity contrast in the internal region.

⇒ **Nusselt number**

$$Nu = Ra_{rh}^{\frac{n+\gamma(2n+2)}{n+2}} \theta^{\beta\frac{2n+2}{n+2}}$$

very close to the classical prediction.

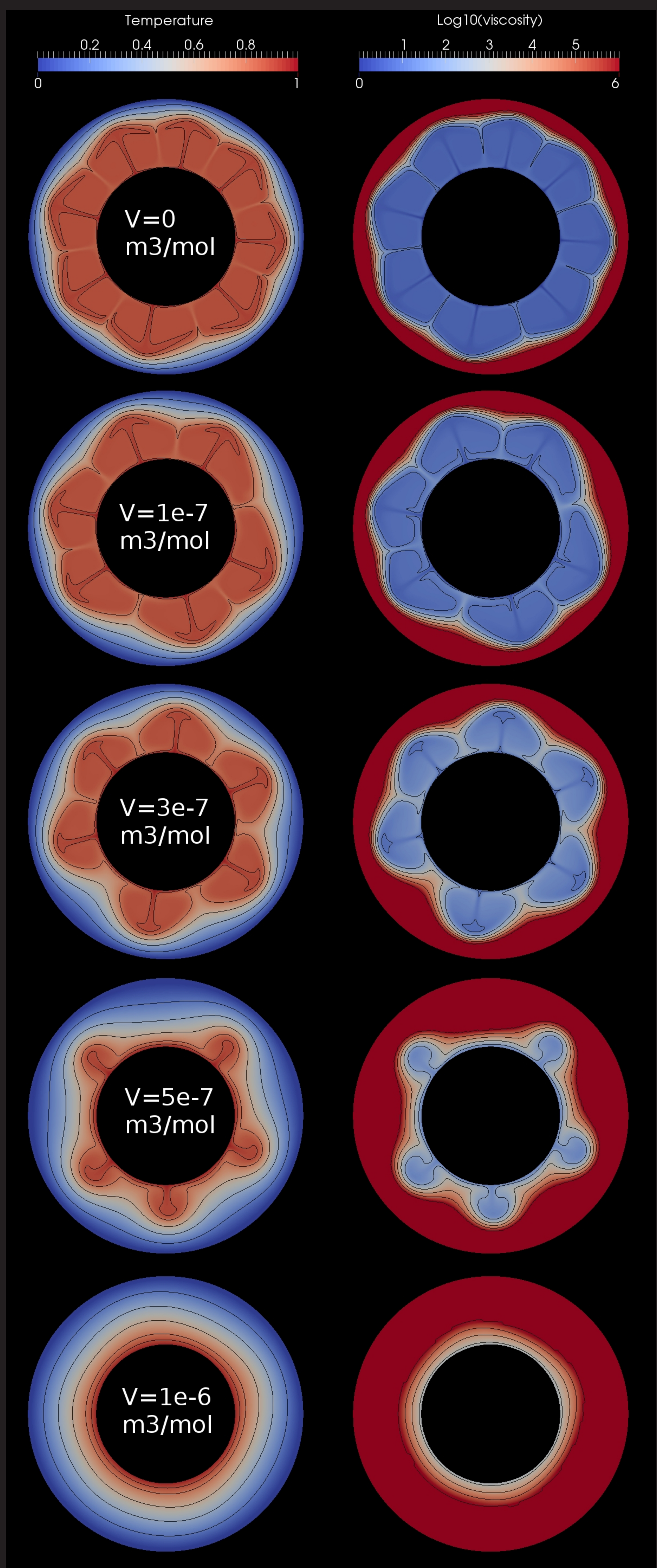
Numerical simulations – Equilibrium state



- 2D Cartesian convection.
- Free-slip BC, bottom heated.
- Pressure-temperature-dependent rheologies.
- Frank-Kamenetskii and Arrhenius formulations.
- Newtonian and non-Newtonian simulations.
- Stationary, unstationary convection.
- Large range of θ and Ra_{rh}.

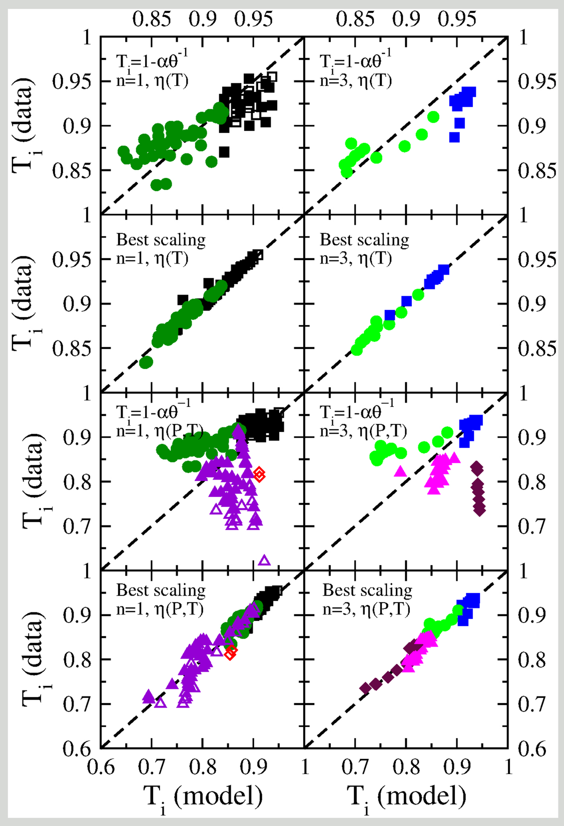
EQUILIBRIUM STATES, NO INTERNAL HEATING

$$\eta(P,T) = 10^{20} \exp\left(\frac{E+PV}{nRT} - \frac{E+P_{300}V}{nRT_b}\right)$$



Results: Equilibrium State

Internal Temperature



– – – – – – $\eta(T)$ – – – – – –

← $T_i = 1 - \frac{\alpha}{\theta}$

← $T_i = 1 - \frac{\alpha}{\theta^{0.90}} \Delta\eta_l^{-0.78}$, (n=1)

← $T_i = 1 - \frac{\alpha}{\theta^{0.92}} \Delta\eta_l^{-1.81}$, (n=3)

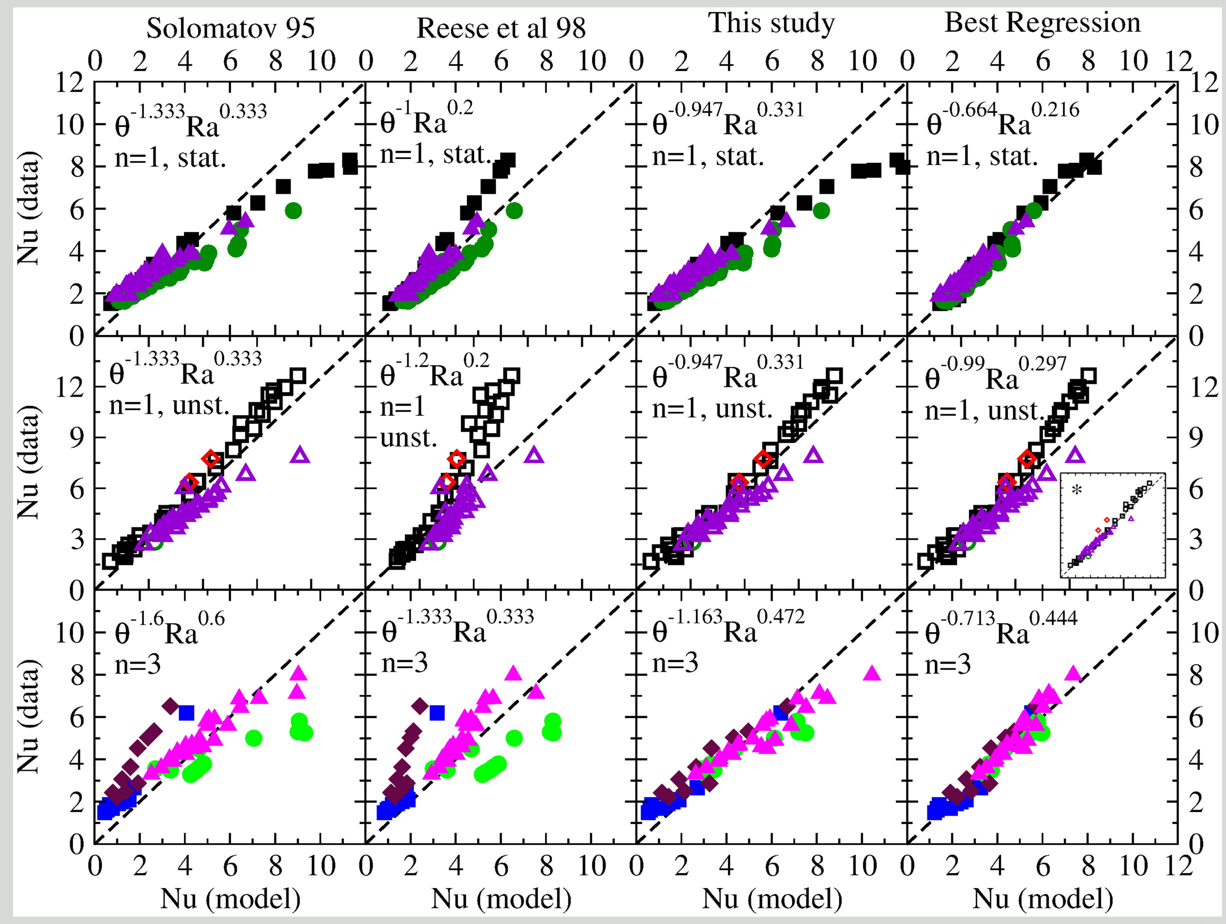
– – – – – – $eta(P,T)$ – – – –

← $T_i = 1 - \frac{\alpha}{\theta}$

← $T_i = 1 - \frac{\alpha}{\theta^{0.72}} (\Delta\eta_l^{-0.64} + \Delta\eta_v^{0.28})$, (n=1)

← $T_i = 1 - \frac{\alpha}{\theta^{0.73}} (\Delta\eta_l^{-1.23} + \Delta\eta_v^{0.39})$, (n=3)

Nusselt Number



⇒ Exponent to θ overestimated in classical scalings $Nu \simeq \theta^{-a} Ra_{rh}^b$

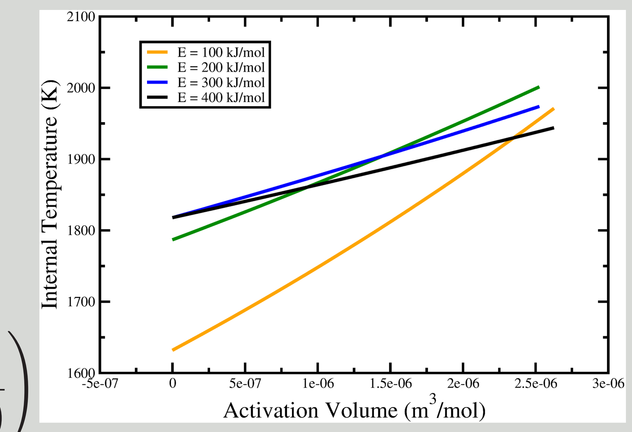
Analytical prediction for fully internally heated planet

INTERNAL HEATING

- Surface heat flux:

$$F_S = H\frac{R}{3}$$

where H is the volumetric density of radiogenic heat sources and R is the radius of the planet.



- Heat flux-Nusselt relation:

$$F_S = 2k\, Nu\, T_i \left(\frac{R^3 - (R-d)^3}{d^2(R^2 + (R-d)^2)}\right)$$

→ pressure-dependent viscosity = hotter planet!

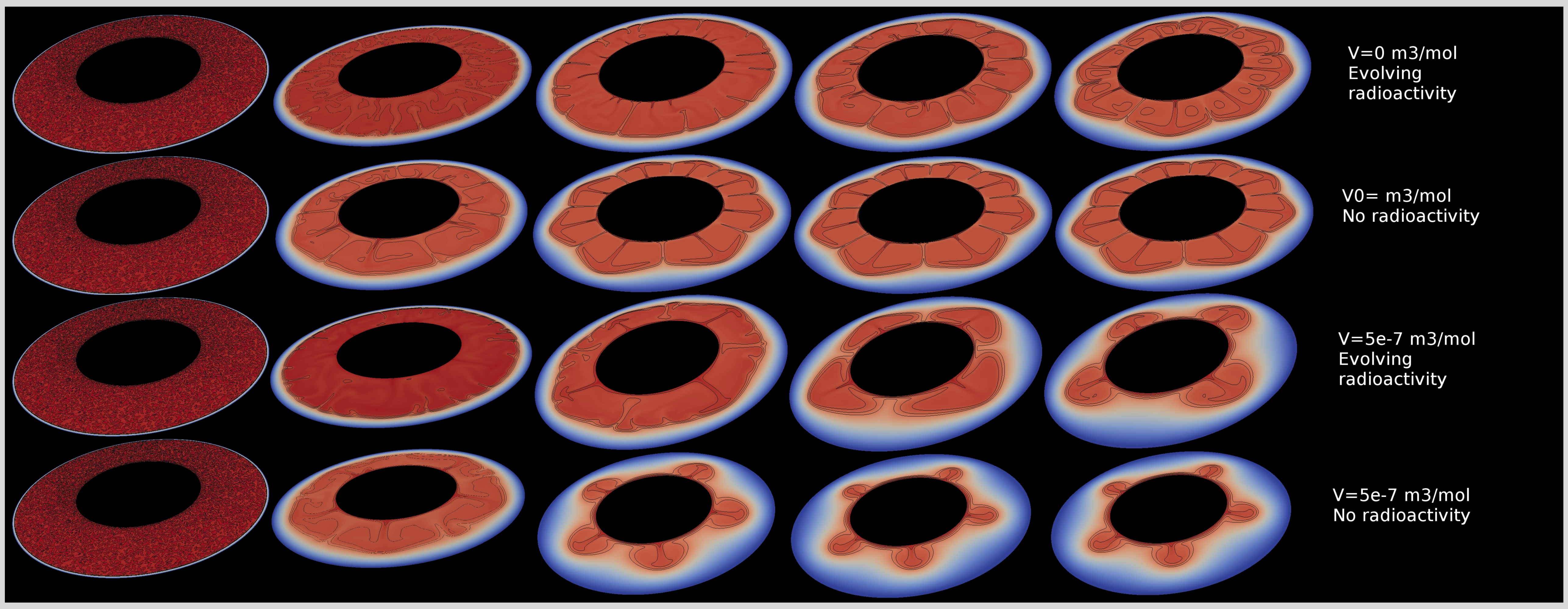
Conclusions

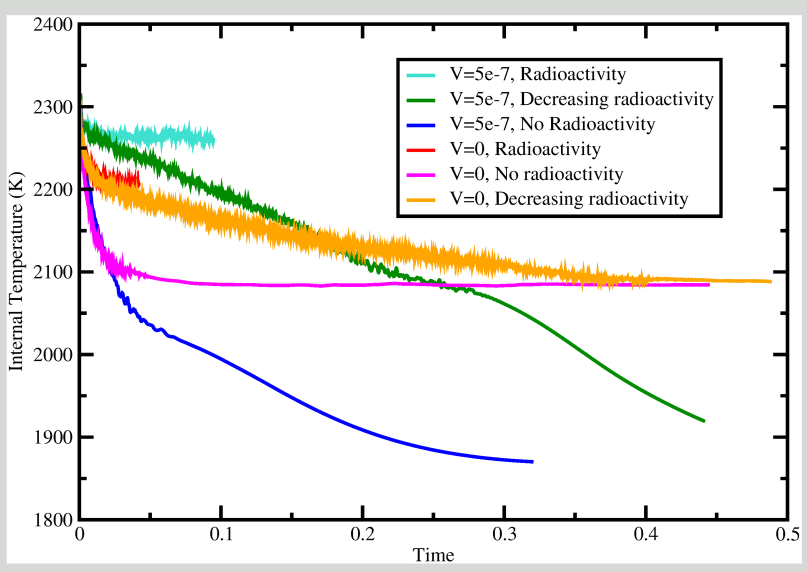
- New model of internal temperature
- Internal viscosity contrasts are important
- Incorporation in boundary layer theory
- Pressure-depent rheologies make internal temperatures more contrasted in time
- Impact of partial melting?

Results: Time-dependent simulations

INTERNAL AND BASAL HEATING

Time ⟶





- Pressure-dependent rheologies makes planets hotter in the initial stage and cooler when radioactivity becomes negligible.