Convection and grain size evolution in mantle and lithosphere of the Earth
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Two phase flow and localization of deformation

FORMALISM OF BERCOCVICI & RICARD 2012
- Grain size evolution for phase i:
  \[ \frac{D R_i}{Dt} = G_i \rho R_i - (1 - \chi_i) R_i^{3/2} \bar{\Psi}_i \]
- Evolution of the curvature between 2 phases:
  \[ \frac{D C}{Dt} = (3/2) \phi_1 \phi_2 G_1 q C_q - \phi_1 \phi_2 C_2 \gamma_i f_i \bar{\Psi}_i \]
- Zener Pining term:
  \[ Z_i = 1 - \chi_2 (1 - \phi_i) R_i^{2/3} \]

Parameter definitions
\[ \begin{align*}
\phi_i & : \text{volume fraction of phase } i \\
R_i & : \text{average grain size (from a full distribution) of phase } i \\
C & : \text{average curvature of grain boundaries between phases.} \\
G_i & : \text{temperature-dependent growth factor of phase } i \\
G_1 & : \text{temperature-dependent growth factor of curvature.} \\
p & : \text{grain growth exponent.} \\
\gamma_i & , \gamma_I & : \text{surface tensions.} \\
f_i G, f_I & : \text{partitioning coefficients} < 1. \\
\bar{\Psi}_i & : \text{mechanical work in dislocation creep in phase } i. \\
\chi_1, \chi_2 & : \text{constants.}
\end{align*} \]

Piezometer – adapted from Bercovici & Ricard (2012 PEPI)

Transition from diffusion to dislocation creep?
Rheological boundaries can be defined within this framework considering strain rate equalities. Perhaps the most important boundary for the mantle is the diffusion-dislocation boundary:
\[ \tau \frac{df}{ds} = \tau \frac{ds}{df} \exp \left( \frac{E_f - E_d}{RT} \right) \]

This stress transition is very parameter-dependent. One can also express this transition in terms of transitional grain size:
\[ d \frac{df}{ds} = \left( \frac{\tau_1 - n A_{df} A_{ds}}{m} \right)^{1/m} \exp \left( \frac{(E_f - E_d)}{mRT} \right) \]

Grain Size in the Mantle?
\[ \begin{align*}
\text{First tests with StagYY in 2D spherical annulus geometry} \\
\text{No phase transitions} \\
\text{Single phase grain size evolution (see Rozel et al. 2011)}
\end{align*} \]

Grain Size in the Mantle? – Phase transitions
\[ \begin{align*}
\text{Simplified phase transitions (Grain size = 5 microns around phase transitions)} \\
\text{Single phase grain size evolution (see Rozel et al. 2011)}
\end{align*} \]

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