10.1098/rspa.2000.0661

Coupled equations for mass and momentum balance in a stream network: theoretical derivation and computational experiments

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Received 24 February 2000; accepted 3 August 2000

In previous work by the authors a rigorous procedure for the derivation of global watershed-scale balance laws for mass, momentum, energy and entropy has been pursued. To complement these, a set of constitutive relations for the closure of the mass and momentum balance equations has also been derived, based on the exploitation of the second law of thermodynamics. In this paper these governing equations, including the constitutive relations, are rederived for the simpler case of the stream channel network of a natural watershed. The derived constitutive relationships for mass and force exchanges amongst channel reaches are physically consistent and thermodynamically admissible insofar as they respect physical constraints and keep the total entropy production of the system always positive. Next, the resulting system of coupled nonlinear ordinary differential equations are simultaneously solved for a natural watershed under realistic conditions. The numerical model presented permits the estimation of space-time fields of average velocity, storage and discharge within all reaches of the network tree during run-off events. The network response, as well as space-time fields of velocity and discharge, are computed for a number of rainfall events of different magnitude and different levels of network discretization. The nonlinearity of the response and the effects of different discretizations of the network are analysed in terms of computational experiments.

Keywords: channel network; balance equations; constitutive relationships; network routing; instantaneous unit response functions

1. Introduction

The purpose of this paper is to develop governing equations for the description of stream channel network responses. The governing equations are based on balances of mass and momentum, are formulated at the spatial scale of a reach, depend only on time, and can be employed for the prediction of space-time fields of average velocity and storage for each reach within the network.

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