WATER RESOURCES RESEARCH, VOL. 36, NO. 7, PAGES 1845–1863, JULY 2000

Conservation equations governing hillslope responses: Exploring the physical basis of water balance

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Abstract. A unifying approach for the derivation of watershed-scale conservation equations governing hydrologic responses has recently been introduced. The approach is based on space-time averaging of the corresponding point-scale conservation equations over an averaging region called the representative elementary watershed (REW). The conservation equations are supplemented by constitutive relationships needed for the closure of various mass and momentum exchange terms. In this paper, we present a summary of the resulting equations for a somewhat simpler problem and show how these equations can be employed to model the long-term water balance of a single, hypothetical REW. The governing equations are formulated in terms of an average saturation and a flow velocity of the unsaturated zone soils and the average thickness of the saturated zone soils. The equations are coupled ordinary differential equations and are nonlinear. Their solution is carried out simultaneously for a variety of combinations of watershed geometry, soil type, and atmospheric forcing. We show how a similarity analysis of the governing equations can lead to a general classification of REWs in terms of meaningful dimensionless similarity variables. In addition, we investigate the long-term water balance and show that the governing equations are able to provide a realistic picture of the impact of changing climate, soil, and topographic controls on long-term water balance.

1. Introduction

Previously, *Reggiani et al.* [1998, 1999a] developed a mathematical framework for watershed thermodynamics with the idea that it could henceforth form the theoretical basis for watershed modeling and measurement. The goal of this paper is to demonstrate, through a simple theoretical exercise, how the above mentioned thermodynamic framework can be employed to model fundamental characteristics of catchment water balance. In this context, we make a series of assumptions with respect to soil properties, system geometry, and atmospheric forcing, which are stated clearly at the outset. For example, spatial variability of soils and climate are ignored, as a first step. Then, through the use of a simple numerical model based on the above theory, the climate, soil, and topographic controls on the annual water budget of a hypothetical watershed entity are explored. We must emphasize that the results thus obtained depend critically on the assumed forms for a number of constitutive relationships that appear in the balance equations and on their assumed parameterizations. These assumed relationships represent preliminary approximations, which require further extension and validation through carefully planned field experiments and fine-scale numerical modeling. The strength of our approach is that it is possible to go back and relax any of these assumptions if deemed necessary for a particular situation.

Paper number 2000WR900066.
0043-1397/00/2000WR900066$09.00

The crucial factors determining the long-term water balance of an unvegetated hillslope or watershed entity can be summarized as topography, soils, and atmospheric forcing. Much of our current understanding of the dynamics of water balance derives from the pioneering work of *Eagleson* [1978a, b, c, d], who investigated climate-soil-vegetation controls on water balance using one-dimensional (vertical) models of infiltration and evapotranspiration. However, Eagleson's work ignored topographic effects controlling both surface runoff by saturation excess and subsurface runoff (drainage). In contrast to Eagleson, *Milly* [1994] investigated process controls on observed spatial patterns of annual water balance but hypothesized that finite storage capacity of soils (not permeability) had the biggest control on water balance. Milly also investigated the effects of seasonality and storminess but did not include the effects of vegetation or its spatial variability. More recently, *Salvucci and Entekhabi* [1995] extended the work of Eagleson by simulating the hydrologic fluxes on idealized hillslopes, by focusing especially on topographic controls. In this case, they used a spatially distributed model of hillslope hydrology, as an extension to Eagleson's work. They worked with a range of soil thicknesses, slope angles, convergence or divergence features, climatic forcing patterns, and soil characteristics. They were able to predict long-term mean or equilibrium water table profiles below the soil surface and net recharge and discharge areas for given topographic, climatic, and pedologic circumstances. However, they too, did not include vegetation effects.

The mathematical framework derived by *Reggiani et al.* [1998, 1999a] comprises the derivation of watershed-scale bal-