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A unifying framework for watershed thermodynamics: constitutive relationships

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Abstract

The balance equations for mass and momentum, averaged over the scale of a watershed entity, need to be supplemented with constitutive equations relating flow velocities, pressure potential differences, as well as mass and force exchanges within and across the boundaries of a watershed. In this paper, the procedure for the derivation of such constitutive relationships is described in detail. This procedure is based on the method pioneered by Coleman and Noll through exploitation of the second law of thermodynamics acting as a constraint-type relationship. The method is illustrated by its application to some common situations occurring in real world watersheds. Thermodynamically admissible and physically consistent constitutive relationships for mass exchange terms among the subregions constituting the watershed (subsurface zones, overland flow regions, channel) are proposed. These constitutive equations are subsequently combined with equations of mass balance for the subregions. In addition, constitutive relationships for forces exchanged amongst the subregions are also derived within the same thermodynamic framework. It is shown that, after linearisation of the latter constitutive relations in terms of the velocity, a watershed-scale Darcy's law governing flow in the unsaturated and saturated zones can be obtained. For the overland flow, a second order constitutive relationship with respect to velocity is proposed for the momentum exchange terms, leading to a watershed-scale Chezy formula. For the channel network REW-scale Saint–Venant equations are derived. Thus, within the framework of this approach new relationships governing exchange terms for mass and momentum are obtained and, moreover, some well-known experimental results are derived in a rigorous manner.

Keywords: Watersheds; Hydrologic theory; Second law of thermodynamics; Mass and force balances; Constitutive relationships; Coleman and Noll procedure

1. Introduction

This work represents the sequel to a previous paper (Reggiani et al. [33]) concerned with the derivation of watershed-scale conservation equations for mass, momentum, energy and entropy. These equations have been derived by averaging the corresponding point scale balance equations over a well defined averaging region called the Representative Elementary Watershed (REW). The REW is a fundamental building block for hydrological analysis, with the watershed being discretised into an interconnected set of REWs, where the stream channel network acts as a skeleton or organising structure. The stream network associated with a watershed is a bifurcating, tree-like structure consisting of nodes inter-connected by channel reaches or links. Associated with each reach or link, there is a well-defined area of the land surface capturing the atmospheric precipitation and delivering it towards the channel reach. These areas uniquely identify the sub-watersheds which we define as REWs. As a result, the agglomeration of the REWs forming the entire watershed resembles the tree-like structure of the channel network on which the discretisation is based, as shown schematically in Fig. 1.

The volume making up a REW is delimited externally by a prismatic mantle, defined by the shape of the ridges circumscribing the sub-watershed. On top, the REW is delimited by the atmosphere, and at the bottom by either an impermeable substratum or an assumed limit depth. The stream reach associated with a given REW can be either a source stream, classified as a *first order*

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