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A unifying framework for watershed thermodynamics: balance equations for mass, momentum, energy and entropy, and the second law of thermodynamics

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The basic aim of this paper is to formulate rigorous conservation equations for mass, momentum, energy and entropy for a watershed organized around the channel network. The approach adopted is based on the subdivision of the whole watershed into smaller discrete units, called representative elementary watersheds (REW), and the formulation of conservation equations for these REWs. The REW as a spatial domain is divided into five different subregions: (1) unsaturated zone; (2) saturated zone; (3) concentrated overland flow; (4) saturated overland flow; and (5) channel reach. These subregions all occupy separate volumina. Within the REW, the subregions interact with each other, with the atmosphere on top and with the groundwater or impermeable strata at the bottom, and are characterized by typical flow time scales.

The balance equations are derived for water, solid and air phases in the unsaturated zone, water and solid phases in the saturated zone and only the water phase in the two overland flow zones and the channel. In this way REW-scale balance equations, and respective exchange terms for mass, momentum, energy and entropy between neighbouring subregions and phases, are obtained. Averaging of the balance equations over time allows to keep the theory general such that the hydrologic system can be studied over a range of time scales. Finally, the entropy inequality for the entire watershed as an ensemble of subregions is derived as constraint-type relationship for the development of constitutive relationships, which are necessary for the closure of the problem. The exploitation of the second law and the derivation of constitutive equations for specific types of watersheds will be the subject of a subsequent paper. © 1998 Elsevier Science Limited. All rights reserved

Keywords: representative elementary watersheds, subregions, balance equations.

NOMENCLATURE

- A mantle surface with horizontal normal delimiting the REW externally
- b external supply of entropy, $[L^2/T^{3o}]$
- C^A external boundary curve of the REW
- C^r length of the channel, $[L]$
- e mass exchange per unit surface area, $[M/TL^2]$
- E internal energy per unit mass, $[L^2/T^2]$
- f external supply term for ψ
- F entropy exchange per unit surface area projection, $[M/T^{3o}]$
- $\boldsymbol{g}$ the gravity vector, $[L/T^2]$
- G production term in the generic balance equation
- h external energy supply, $[L^2/T^3]$
- $\mathbf{i}$ general flux vector of ψ
- $\mathbf{j}$ microscopic non-convective entropy flux $[M/T^{3o}]$
- L rate of net production of entropy, $[M/T^{3o}L]$
- m^r volume per unit channel length, equivalent to the average cross-sectional area, $[L^3/L]$
- M number of REWs making up the entire watershed
- $\mathbf{n}^{jA}$ unit normal pointing from the j-subregion outward with respect to the mantle surface
- $\mathbf{n}^j$ unit normal pointing from the j-subregion into the atmosphere or into the ground

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