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Boundary and Interface Conditions in Porous Media

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Correct specification of conditions at macroscopic surfaces of discontinuity and along the boundary is essential to the complete mathematical description of flow and transport in porous media. This work provides the framework for systematic derivation of appropriate conditions. It thereby allows for generalization of the conditions commonly in use and exposes the assumptions which underlie them. General balance equations for a zone separating two regions are simplified to the case where the regions are in good thermodynamic contact. By application, interface conditions for balance of mass, chemical species, momentum, and energy are obtained for (1) adjacent porous media containing the same fluid, (2) water flooding of an oil reservoir, (3) a salt water/freshwater interface, and (4) the boundary between a surface water body and the porous medium.

INTRODUCTION

An essential part of the mathematical description of any transport problem in porous media is the correct specification of boundary conditions. External boundaries of a medium can be considered as interfaces between the porous medium and the outside world. Also, when different porous media neighboring each other are to be modeled, interface conditions which relate thermodynamic properties of the adjacent media are required. Such relations are commonly called "jump conditions." Both external and internal boundaries are considered here within the same general framework.

Interfaces may be classified into two general types: simple and nonsimple. A simple interface is the locus of a macroscopic surface of discontinuity in solid and/or fluid properties such as an abrupt change in permeability or porosity. Generally speaking, simple interfaces arise in the macroscopization or volume averaging process. A nonsimple interface is the macroscopic representation of a physical transition zone between two porous media; for example, a clay lens between two aquifers or a fracture filled with permeable material. A nonsimple interface is modeled as being important only insofar as it facilitates or restricts interaction between porous media on each side.

Interface conditions for simple interfaces in porous media have been studied by *Raats* [1972*a*, *b*], *Bachmat and Bear* [1972], *Bear* [1979], *Hassanizadeh and Gray* [1979], and *Bear and Verruijt* [1987], among others. The interface condition for a macroscopic thermodynamic property of phase α, $\bar{\psi}^\alpha$, typically has the following form:

$$[\![\varepsilon^\alpha \langle\rho\rangle^\alpha \bar{\psi}^\alpha(\bar{\mathbf{v}}^\alpha - \mathbf{W}) - \mathbf{i}^\alpha]\!] \cdot \mathbf{N} = f_b^\alpha \qquad (1)$$

where boldface brackets denote the jump of the bracketed quantity across the interface and f_b^α is the supply of $\bar{\psi}^\alpha$ to the phase α by other phases present in the interface. However, most authors have assumed that the interface itself is devoid of thermodynamic properties and have set f_b^α to zero (see for example, *Muller* [1971], *Raats* [1972*a*], *Bachmat and Bear* [1972], *Bowen and Doria* [1973], and *Hassanizadeh and Gray* [1979]). *Raats* [1972*b*] has suggested an interface condition similar to the one above and has applied it to a surface of discontinuity in an unsaturated rigid porous medium. However, he sets f_b^α identically to zero for the α phase momentum balance and introduces a parameter λ which must be manipulated somewhat arbitrarily in order to obtain a correct jump condition and balance equation. Recently, *Bear and Verruijt* [1987], have also employed interface condition (1) with a nonzero f_b^α and have applied it to a number of different interfaces. To complete the description of the interface, they also postulate the existence of a state of "thermodynamic equilibrium" between the two porous media at the interface. They require the continuity of pressure, mass density, and temperature of the fluid phase and the mass concentration of species at the interface [*Bear and Verruijt*, 1987, p. 67]. However, this requirement is overly restrictive. For example, continuity of mass density and species mass concentration does not hold in a number of practical situations.

Recently, *Hassanizadeh and Gray* [1989] have developed general equations of balance at interfaces in a multiphase system. These equations are applicable to both simple and nonsimple interfaces. Two examples of nonsimple interfaces are treated by *Hassanizadeh and Gray* [1989].

In the present work, a number of cases of practical interest involving simple interfaces are considered and appropriate interface conditions are derived. This work provides a fundamental theoretical basis for interface conditions normally employed in practice and explicitly exposes various assumptions underlying them. The procedure is such that in the absence of these assumptions, more general conditions for more complicated situations are obtained. The development presented here also provides insight to the theoretical description of processes at interfaces within a porous medium.

GENERAL BALANCE EQUATIONS FOR SIMPLE INTERFACES

The general form of the conservation equation for an interface derived by *Hassanizadeh and Gray* [1989] reads

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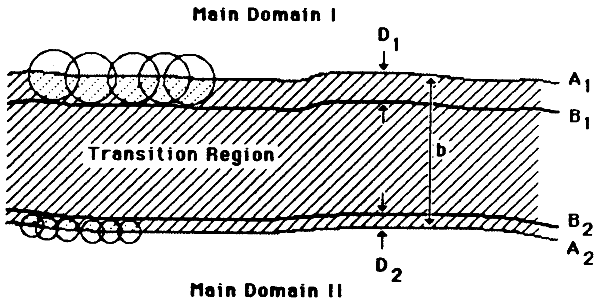

Fig. 1. Nonsimple interface of thickness b between two main domains.

$$\frac{\partial}{\partial t}(b\varepsilon^{b\alpha}\langle\rho\rangle^{b\alpha}\bar{\psi}^{b\alpha}) + \nabla'\cdot(b\varepsilon^{b\alpha}\langle\rho\rangle^{b\alpha}\bar{\mathbf{v}}'^{b\alpha}\bar{\psi}^{b\alpha})$$
$$- \nabla'\cdot(b\bar{\mathbf{i}}'^{b\alpha}) - b\varepsilon^{b\alpha}\langle\rho\rangle^{b\alpha}\bar{f}^{b\alpha}$$
$$- \frac{1}{\delta S}\sum_{\beta\neq\alpha}\int_{\delta A^b_{\alpha\beta}}[\rho\psi(\mathbf{w}-\mathbf{v})+\mathbf{i}]\cdot\mathbf{n}^{\alpha\beta}\,da$$
$$= [\varepsilon^\alpha\langle\rho\rangle^\alpha\bar{\psi}^\alpha(\bar{\mathbf{v}}^\alpha - \mathbf{W}) - \bar{\mathbf{i}}^\alpha]|_1\cdot\mathbf{N}_1$$
$$+ [\varepsilon^\alpha\langle\rho\rangle^\alpha\bar{\psi}^\alpha(\bar{\mathbf{v}}^\alpha - \mathbf{W}) - \bar{\mathbf{i}}^\alpha]|_2\cdot\mathbf{N}_2 \qquad (2)$$

subject to

$$\sum_\alpha\sum_{\beta\neq\alpha}\int_{\delta A^b_{\alpha\beta}}[\rho\psi(\mathbf{w}-\mathbf{v})+\mathbf{i}]\cdot\mathbf{n}^{\alpha\beta}\,da = 0 \qquad (3)$$

Various quantities appearing in these equations are described in the Notation. For phase α existing within a macroscopic interface, (2) relates the transport of some property within the interface to the exchange of this property with the α phase present in the main domains. The terms on the left side of (2) describe processes occurring within the interface, while the terms on the right side describe transport to the interface from the α phase of the adjacent main domains. Equation (3) provides a constraint on interphase transport within the interface. The equations are valid for simple as well as nonsimple interfaces.

A nonsimple interface is an intermedia zone in the porous medium (Figure 1) which attains its own thermodynamic properties (e.g., a fracture or a thin clay lens). For such interfaces, the thickness b is larger than $D_1 + D_2$. All terms in (2) may be of importance and have to be taken into account. A simple interface (Figure 2), on the other hand, is a narrow zone of transition between different media or fluids with thickness on the order of an averaging volume length scale. This narrow zone of transition is introduced as a byproduct of the macroscopization process [see *Hassanizadeh and Gray*, 1989]. The transport within a simple interface may be neglected. However, solid and/or fluid phase properties of neighboring main domains may undergo appreciable changes across these interfaces. An example is the interface between two porous media with different porosity, permeability, and other properties.

Because simple interfaces are considered to be narrow transition zones with thickness of the order of the representative elementary volume (REV) size, those terms in (2) which are multiplied by b may be neglected. Thus with appropriate selection of the general properties for (2) (see, for example, Table 1 in the work by *Hassanizadeh and Gray* [1989]) the following set of particular balance equations are obtained for simple interfaces. Note that to minimize notation, the averaging signs, the overbars and angle brackets, have been omitted in subsequent equations.

Mass

$$[\![\varepsilon^\alpha\rho^\alpha(\mathbf{v}^\alpha - \mathbf{W})]\!]\cdot\mathbf{N} = -\frac{1}{\delta S}\sum_{\beta\neq\alpha}\int_{\delta A^b_{\alpha\beta}}[\rho(\mathbf{w}-\mathbf{v})]\cdot\mathbf{n}^{\alpha\beta}\,da \qquad (4)$$

Mass of species

$$[\![\varepsilon^\alpha\rho^\alpha\omega^{a\alpha}(\mathbf{v}^\alpha - \mathbf{W}) - \mathbf{j}^{a\alpha}]\!]\cdot\mathbf{N}$$
$$= -\frac{1}{\delta S}\sum_{\beta\neq\alpha}\int_{\delta A^b_{\alpha\beta}}[\rho\omega^a(\mathbf{w}-\mathbf{v})+\mathbf{j}^a]\cdot\mathbf{n}^{\alpha\beta}\,da \qquad (5)$$

Momentum

$$[\![\varepsilon^\alpha\rho^\alpha\mathbf{v}^\alpha(\mathbf{v}^\alpha - \mathbf{W}) - \mathbf{t}^\alpha]\!]\cdot\mathbf{N}$$
$$= -\frac{1}{\delta S}\sum_{\beta\neq\alpha}\int_{\delta A^b_{\alpha\beta}}[\rho\mathbf{v}(\mathbf{w}-\mathbf{v})+\mathbf{t}]\cdot\mathbf{n}^{\alpha\beta}\,da \qquad (6)$$

Energy

$$[\![\varepsilon^\alpha\rho^\alpha(E^\alpha + \tfrac{1}{2}\mathbf{v}^\alpha\cdot\mathbf{v}^\alpha)(\mathbf{v}^\alpha - \mathbf{W}) - \mathbf{t}^\alpha\cdot\mathbf{v}^\alpha - \mathbf{q}^\alpha]\!]\cdot\mathbf{N}$$
$$= -\frac{1}{\delta S}\sum_{\beta\neq\alpha}\int_{\delta A^b_{\alpha\beta}}\{\rho(E+\tfrac{1}{2}\mathbf{v}\cdot\mathbf{v})(\mathbf{w}-\mathbf{v})+\mathbf{t}\cdot\mathbf{v}+\mathbf{q}\}\cdot\mathbf{n}^{\alpha\beta}\,da \qquad (7)$$

The boldface brackets in these equations denote the jump of quantities across the interface given by

$$[\![a\mathbf{f}]\!]\cdot\mathbf{N} = (a\mathbf{f})_1\cdot\mathbf{N}_1 + (a\mathbf{f})_2\cdot\mathbf{N}_2$$
$$= [(a\mathbf{f})_1 - (a\mathbf{f})_2]\cdot\mathbf{N}_1 = [(a\mathbf{f})_2 - (a\mathbf{f})_1]\cdot\mathbf{N}_2 \qquad (8)$$

where a and $\mathbf{f}$ are considered as typical scalar and vector variables. Note that the terms on the right-hand side of (4)–(7) will not necessarily be small even for small b. The magnitudes of these terms are dependent upon the size of $\alpha\beta$ interfacial areas within the transition zone as well as the types of processes which take place there. In fact, these terms are essential for obtaining correct interface conditions in many situations. For example, it can be shown that with the right-hand side of (6) neglected, the pressure will be predicted to undergo a jump at an interface between two porous media with different porosities but saturated with the same fluid, a condition which is known to be incorrect. Also, in the case of a moving reaction front in the porous medium,

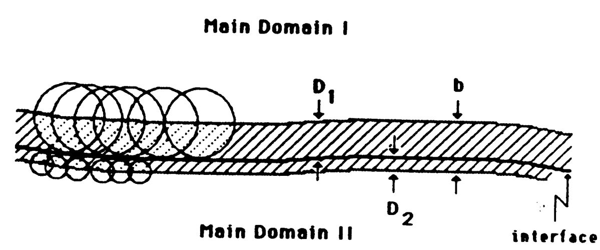

Fig. 2. Simple interface with length scale of the order of an averaging volume between two domains.

the term on the right-hand side of (5) accounts for the mass exchange between the fluid and solid which often is assumed to take place only at the interface. However, it is important to make use of the fact deriving from (3) that the sum over all phases of the right-hand side in (4)–(7) is zero. Appropriate constitutive equations need to be given for the indicated integrals based on the nature of the interface and the kind of processes being considered.

In this article, (4)–(7) are employed to derive appropriate conditions at a number of commonly encountered interfaces in porous media problems.

Characterization of Simple Interfaces

Balance equations are not sufficient for full thermodynamic description of interfaces. Similarly to a single-phase or multiphase body, conservations laws at an interface must be supplemented with constitutive relations which account for the intrinsic characteristics of the interface. Examples of constitutive equations for nonsimple interfaces have been given in the work by *Hassanizadeh and Gray* [1989].

Before constitutive assumptions for simple interfaces can be defined, the characteristics assigned to such interfaces are summarized.

1. Simple interfaces are narrow transition zones with the thickness of the order of the REV length scale. Therefore transport and storage of properties in the "plane" of the interface may be neglected.

2. Simple interfaces are not actual physical barriers between two adjacent media and therefore they do not offer any resistance to the transport of thermodynamic properties normal to the interface.

3. The media on the two sides of the interface are in direct thermodynamic contact.

Because of these characteristics, it is reasonable to assume that the two adjacent media at the interface exist at a state of "thermodynamic equilibrium." The conditions for such an equilibrium are [*Denbigh*, 1981, sec. 2.9)

$$p_1^\alpha = p_2^\alpha \tag{9a}$$

$$T_1^\alpha = T_2^\alpha \tag{9b}$$

$$\mu_1^{a\alpha} = \mu_2^{a\alpha} \tag{9c}$$

where p^α is the thermodynamic pressure of phase α, T^α is the temperature of phase α, and $\mu^{a\alpha}$ is the chemical potential of species a in phase α. The first of these relations will be proven later with the help of the principle of momentum conservation. Note that the last relation does not necessarily imply the continuity of mass fraction of species a at the interface. In fact, if the fluids at the two sides of the interface are different, at equilibrium (see, for example, *Bird et al.* [1960])

$$\omega_1^{a\alpha} = f(\omega_2^{a\beta}) \tag{10}$$

Also note that continuity of fluid mass density may not be imposed identically when different fluids exist on the two sides of the interface. Fluid densities are obtained from equations of state as a function $\rho^\alpha = \rho^\alpha(p^\alpha, T^\alpha, \omega^{a\alpha})$.

It should be emphasized that equalities (9) and (10) hold only if the media at the two sides of the interface are in "good" thermodynamic contact. For example, in the case of a nonsimple interface, the above relations do not hold. Instead, constitutive equations for the flux of mass, heat, and chemical species must be provided [*Hassanizadeh and Gray*, 1989].

Two Porous Media Containing the Same Fluid

Consider two adjacent porous media with different porosities and permeabilities which are therefore separated by a simple transition interface which allows for thermomechanical connection. The two media are assumed to contain the same fluid, but the flow may be unsaturated (i.e., a gas phase may also be present.) Assume that mass and momentum exchanges between the gas phase and the other phases is negligible. The notation $\alpha = f$, s, and g for the fluid, solid, and gas phases, respectively, is adopted.

Mass Balance

From (4), the equations of mass balance at the interface for the fluid and solid phases are

$$[\varepsilon s^f \rho^f(\mathbf{v}^f - \mathbf{W})] \cdot \mathbf{N} = -\frac{1}{\delta S}\int_{\delta A_{fs}^b} \rho(\mathbf{w} - \mathbf{v}) \cdot \mathbf{n}^{fs}\, da \tag{11}$$

$$[(1-\varepsilon)\rho^s(\mathbf{v}^s - \mathbf{W})] \cdot \mathbf{N} = -\frac{1}{\delta S}\int_{\delta A_{sf}^b} \rho(\mathbf{w} - \mathbf{v}) \cdot \mathbf{n}^{sf}\, da \tag{12}$$

respectively, where ε is the porosity and s^f is the saturation of the fluid. The right-hand sides of (11) and (12) neglect mass exchange with the gas phase but account for the net exchange rate of mass between the fluid and solid phases within the transition zone. In general, a constitutive equation for this term must be provided, such as the expression of *Nguyen et al.* [1982]. Furthermore, (3) requires that all material leaving one phase must be taken up by another phase such that the terms on the right of (11) and (12) are equal in magnitude but opposite in sign with

$$\int_{\delta A_{fs}^b} \rho(\mathbf{w} - \mathbf{v}) \cdot \mathbf{n}^{fs}\, da + \int_{\delta A_{sf}^b} \rho(\mathbf{w} - \mathbf{v}) \cdot \mathbf{n}^{sf}\, da = 0 \tag{13}$$

Define the Darcy velocity of the fluid as

$$\mathbf{u}^f = \varepsilon s^f(\mathbf{v}^f - \mathbf{v}^s) \tag{14}$$

and the medium bulk density (with the density of the gas phase neglected) by

$$\rho^m = \varepsilon s^f \rho^f + (1-\varepsilon)\rho^s \tag{15}$$

Addition of (11) and (12) in light of restriction (13) and some algebra yields

$$[\rho^f \mathbf{u}^f] \cdot \mathbf{N} + [\rho^m(\mathbf{v}^s - \mathbf{W})] \cdot \mathbf{N} = 0 \tag{16}$$

In general, (12) and (16) and a constitutive relation for the integral in (12) comprise the set of boundary conditions which must supplement the differential equations for the main domains. However, in many practical situations, assumptions may be made which allow for additional simplification of these equations.

An important situation is where no phase exchange occurs between the fluid and solid or when the reacting components exist at such low concentrations that chemical processes do not affect the mass balance and the dynamics of the phases. In these cases, the right-hand side of (12) may be neglected.

If, in addition, the solid phase grains do not cross the interface boundary (i.e., if the interface acts as a material surface for the solid phase), then regardless of the motion of the boundary,

$$(\mathbf{v}^s - \mathbf{W})|_1 \cdot \mathbf{N}_1 = (\mathbf{v}^s - \mathbf{W})|_2 \cdot \mathbf{N}_2 = 0 \tag{17}$$

As a result, (16) takes the well-known form of the jump condition for mass flux commonly employed in porous media flow

$$[\rho^f \mathbf{u}^f] \cdot \mathbf{N} = 0 \tag{18}$$

In a large class of groundwater flow problems, a more restricted relation is used (see, for example, *Bear* [1979]), that is,

$$[\mathbf{u}^f] \cdot \mathbf{N} = 0 \tag{19}$$

This condition will be valid if the value of mass density is continuous, i.e.,

$$[\rho^f] = 0 \tag{20}$$

Because normally ρ^f is a known function of fluid pressure, temperature, and species concentrations, only when the continuity of these three variables at the interface can be assumed may (19) be employed. Such a condition is met, for example, in isothermal flow of an incompressible fluid containing low concentrations of dissolved species. Equation (19), together with Darcy's law and the condition of continuity of pressure form the basis for describing the refraction and incidence of flow at a boundary between two porous media (see, for example, *Raats* [1972*b* and 1973]; and *Irmay* [1964]).

Mass Balance for Species

From (5), the following relation for component $a(a = 1, \cdots, N)$ of the fluid phase is obtained:

$$[\varepsilon s^f \rho^f \omega^{af}(\mathbf{v}^f - \mathbf{W}) - \mathbf{j}^{af}] \cdot \mathbf{N}$$

$$= -\frac{1}{\delta S}\int_{\delta A^b_{fs}} \{\rho\omega^a(\mathbf{w} - \mathbf{v}) + \mathbf{j}^a\} \cdot \mathbf{n}^{fs}\, da = -R^a_{\text{het}} \tag{21}$$

subject to

$$\sum_a R^a_{\text{het}} = \frac{1}{\delta S}\int_{\delta A^b_{fs}} \rho(\mathbf{w} - \mathbf{v}) \cdot \mathbf{n}^{fs}\, da \tag{22}$$

Terms on the right-hand side of (21) are the net rate of transport of mass of component a to the solid phase as a result of phase change. Note that a term due to mass exchange resulting from homogeneous reactions (between component a and other components of the fluid phase) is not present, because for small values of b, such a term is negligible. Combined with appropriate constitutive equations for R^a_{het} and $\mathbf{j}^{af}$, (21) is the most general interface condition for fluid components between two different porous media.

If the interface boundaries are material surfaces for the solid phase, then (17) may be employed to simplify (21) to the form

$$[\rho^f \omega^{af} \mathbf{u}^f - \mathbf{j}^{af}] \cdot \mathbf{N} = -R^a_{\text{het}} \tag{23}$$

Employing a Fickian-type relation for $\mathbf{j}^{af}$, and acknowledging condition (18), one may express (23) as

$$(\omega^{af}_1 - \omega^{af}_2)(\rho^f \mathbf{u}^f)|_1 \cdot \mathbf{N}_1 - [\rho^f \mathbf{D}^a \cdot \nabla\omega^{af}] \cdot \mathbf{N} = -R^a_{\text{het}} \tag{24}$$

where $\mathbf{D}^a$ is the dispersion tensor for species a.

Because the same fluid is present on both sides, the equilibrium condition (10) simply becomes $\omega^{af}_1 = \omega^{af}_2$. If, additionally no heterogeneous reactions take place at the interface, (24) reduces to

$$[\rho^f \mathbf{D} \cdot \nabla\omega^{af}] \cdot \mathbf{N} = 0 \tag{25}$$

Equations (10) and (24) (or $\omega^{af}_1 = \omega^{af}_2$ plus equation (25)) provide a complete set of interface conditions for mass transport problems.

Momentum Balance

The equation of momentum balance (6) for a single-fluid system yields

$$[\varepsilon s^f \rho^f \mathbf{v}^f(\mathbf{v}^f - \mathbf{W}) - \mathbf{t}^f] \cdot \mathbf{N}$$

$$= -\frac{1}{\delta S}\int_{\delta A^b_{fs}} \{\rho\mathbf{v}(\mathbf{w} - \mathbf{v}) + \mathbf{t}\} \cdot \mathbf{n}^{fs}\, da \tag{26}$$

$$[(1-\varepsilon)\rho^s \mathbf{v}^s(\mathbf{v}^s - \mathbf{W}) - \mathbf{t}^s] \cdot \mathbf{N}$$

$$= -\frac{1}{\delta S}\int_{\delta A^b_{sf}} \{\rho\mathbf{v}(\mathbf{w} - \mathbf{v}) + \mathbf{t}\} \cdot \mathbf{n}^{sf}\, da \tag{27}$$

The terms on the right-hand side of (26) and (27) account for the exchange of momentum between fluid and solid phases within the transition zone as a result of chemical and mechanical interactions. Interactions with the gas phase have been neglected. Multiplication of (11) by the interface velocity $\mathbf{W}$ and subtraction from (26) yields

$$[\varepsilon s^f \rho^f(\mathbf{v}^f - \mathbf{W})(\mathbf{v}^f - \mathbf{W}) - \mathbf{t}^f] \cdot \mathbf{N}$$

$$= -\frac{1}{\delta S}\int_{\delta A^b_{fs}} \{\rho\bar{\mathbf{v}}^{bf}(\mathbf{w} - \mathbf{v}) + \mathbf{t}\} \cdot \mathbf{n}^{fs}\, da \tag{28}$$

where

$$\bar{\mathbf{v}}^{bf} = \mathbf{v} - \mathbf{W} \tag{29}$$

Also, instead of (27), the sum of (26) and (27) will be used

$$[\varepsilon s^f \rho^f \mathbf{v}^f(\mathbf{v}^f - \mathbf{W}) + (1-\varepsilon)\rho^s \mathbf{v}^s(\mathbf{v}^s - \mathbf{W}) - \mathbf{t}^f - \mathbf{t}^s] \cdot \mathbf{N} = 0 \tag{30}$$

To apply these last three equations, appropriate constitutive relations for $\mathbf{t}^f$, $\mathbf{t}^s$ and the integral on the right-hand side of (28) are needed.

After *Hassanizadeh and Gray* [1989], the following constitutive relations are appropriate for a simple interface:

$$\mathbf{t}^f = -\varepsilon s^f p^f \mathbf{I} \tag{31}$$

$$\mathbf{t}^s = -(1-\varepsilon)p^f \mathbf{I} + {}_e\mathbf{t}^s \tag{32}$$

$$\frac{1}{\delta S}\int_{\delta A^b_{fs}} \{\rho\bar{\mathbf{v}}^{bf}(\mathbf{w} - \mathbf{v}) + \mathbf{t}\} \cdot \mathbf{n}^{fs}\, da$$

$$= -[\varepsilon s^f p^f]\mathbf{N} + \varepsilon^b s^{bf}[p^f]\mathbf{N} \tag{33}$$

where p^f is the thermodynamic pressure of the fluid within the main domains, ${}_e\mathbf{t}^s$ is the solid phase effective stress, and ε^b is the porosity of the interface. Note that as is normal practice in the description of porous media flow, the fluid has been assumed to be inviscid at the macroscale. In the constitutive equation for the solid-fluid drag force, equation (33), only an equilibrium part is included. That is, the resistance of the simple interface to the flow is assumed negligible because the thickness of the transition zone is equal to the REV scale.

In most practical cases of interest in porous media flow, the macroscopic advective fluxes of momentum may be neglected. Thus the velocity terms in the left side of (28) and (30) may be dropped such that after substitution of (31)–(33) these equations become

$$\varepsilon^b s^{bf}[p^f]\mathbf{N} = 0 \tag{34}$$

$$[p^f]\mathbf{N} - [{}_e\mathbf{t}^s] \cdot \mathbf{N} = 0 \tag{35}$$

The inner product of (34) with $\mathbf{N}_1$ yields the commonly invoked boundary condition at a surface of discontinuity within a porous medium:

$$p_1^f = p_2^f \tag{36}$$

The present derivation of (36) seems to be the first based rigorously on the principle of momentum conservation wherein the underlying assumptions are exposed. Note that neglect of the solid-fluid force (the right-hand side of equation (28)) will lead to the condition $(\varepsilon s^f p^f)_1 = (\varepsilon s^f p^f)_2$ which is incorrect when $\varepsilon_1 s_1^f \neq \varepsilon_2 s_2^f$. Equations (18) or (19) and (36) provide the complete set of interface conditions for flow problems.

Next, examine the solid phase interface condition (35) which, when (34) applies, reduces to

$$[{}_e\mathbf{t}^s] \cdot \mathbf{N} = 0 \tag{37}$$

This equation states the continuity of traction at the interface. Often, it is necessary to formulate this condition in terms of the medium displacement vector $\mathbf{U}$. Employment of a linear relation for ${}_e\mathbf{t}^s$ (see, for example, *Hassanizadeh and Gray* [1980] for isotropic porous media in (37) yields

$$[{}_e\mathbf{t}^s] \cdot \mathbf{N} = [\mu^s(\nabla\mathbf{U} + (\nabla\mathbf{U})^T) + \beta^s(\nabla \cdot \mathbf{U})\mathbf{I}] \cdot \mathbf{N} = 0 \tag{38}$$

where μ^s and β^s are the soil elastic constants.

Energy Balance

The equations of energy balance at the boundary, given by (7), for the system under study are

$$[\varepsilon s^f \rho^f(E^f + \tfrac{1}{2}\mathbf{v}^f \cdot \mathbf{v}^f)(\mathbf{v}^f - \mathbf{W}) - \mathbf{t}^f \cdot \mathbf{v}^f - \mathbf{q}^f] \cdot \mathbf{N}$$
$$= -\frac{1}{\delta S}\int_{\delta A_{fs}^b} \{\rho(E + \tfrac{1}{2}\mathbf{v} \cdot \mathbf{v})(\mathbf{w} - \mathbf{v}) + \mathbf{t} \cdot \mathbf{v} + \mathbf{q}\} \cdot \mathbf{n}^{fs}\, da$$
$$-\frac{1}{\delta S}\int_{\delta A_{fg}^b} \mathbf{q} \cdot \mathbf{n}^{fg}\, da \tag{39}$$

$$[(1 - \varepsilon)\rho^s(E + \tfrac{1}{2}\mathbf{v}^s \cdot \mathbf{v}^s)(\mathbf{v}^s - \mathbf{W}) - \mathbf{t}^s \cdot \mathbf{v}^s - \mathbf{q}^s] \cdot \mathbf{N}$$
$$= -\frac{1}{\delta S}\int_{\delta A_{sf}^b} \{\rho(E + \tfrac{1}{2}\mathbf{v} \cdot \mathbf{v})(\mathbf{w} - \mathbf{v}) + \mathbf{t} \cdot \mathbf{v} + \mathbf{q}\} \cdot \mathbf{n}^{sf}\, da$$
$$-\frac{1}{\delta S}\int_{\delta A_{sg}^b} \mathbf{q} \cdot \mathbf{n}^{sg}\, da \tag{40}$$

The first term inside the first integral on the right-hand side of (39) or (40) accounts for energy exchange between fluid and solid phases as a result of mass exchange. If the mass exchange is negligible, this term may be neglected. The other two terms in each of these integrals account for energy exchange as a result of thermomechanical interactions. The last integrals in (39) and (40) account for thermal interaction with the gas phase when mass and momentum exchange are negligible. The following restriction, which follows directly from (3), applies to these terms for the case of negligible mass exchange:

$$\frac{1}{\delta S}\int_{\delta A_{fs}^b} (\mathbf{t} \cdot \mathbf{v}) \cdot \mathbf{n}^{fs}\, da + \frac{1}{\delta S}\int_{\delta A_{sf}^b} (\mathbf{t} \cdot \mathbf{v}) \cdot \mathbf{n}^{sf}\, da$$
$$+\frac{1}{\delta S}\sum_{\alpha}\sum_{\beta \neq \alpha} \int_{\delta A_{\alpha\beta}^b} \mathbf{q} \cdot \mathbf{n}^{\alpha\beta}\, da = 0 \tag{41}$$

where α and β take on values of s, f, and g. Constitutive relations are needed for terms in (39) and (40) as well as for the integrals.

In the absence of mass exchange, the first integral on the right side of (39) may be rearranged to form

$$\frac{1}{\delta S}\int_{\delta A_{fs}^b} (\mathbf{t} \cdot \mathbf{v} + \mathbf{q}) \cdot \mathbf{n}^{fs}\, da = \left[\frac{1}{\delta S}\int_{\delta A_{fs}^b} \mathbf{t} \cdot \mathbf{n}^{fs}\, da\right] \cdot \mathbf{W}$$
$$+\frac{1}{\delta S}\int_{\delta A_{fs}^b} (\mathbf{t} \cdot \tilde{\mathbf{v}}^{bf} + \mathbf{q}) \cdot \mathbf{n}^{fs}\, da \tag{42}$$

where $\tilde{\mathbf{v}}^{bf}$ is defined by (29). For the first integral on the right-hand side of (42), (33) may be employed. For the second term, the following relation is proposed:

$$\frac{1}{\delta S}\int_{\delta A_{fs}^b} (\mathbf{t} \cdot \tilde{\mathbf{v}}^{bf} + \mathbf{q}) \cdot \mathbf{n}^{fs}\, da = \lambda^{fs}(T^{bf} - T^{bs}) \tag{43}$$

where λ is a heat transfer coefficient and T^{bs} and T^{bf} are the solid and fluid temperatures in the boundary. Substitution from (33) and (43) into (42), yields

$$\frac{1}{\delta S}\int_{\delta A_{fs}^b} (\mathbf{t} \cdot \mathbf{v} + \mathbf{q}) \cdot \mathbf{n}^{fs}\, da$$
$$= -([\varepsilon s^f p^f] - \varepsilon^b s^{bf}[p^f])\mathbf{W} \cdot \mathbf{N} + \lambda^{fs}(T^{bf} - T^{bs}) \tag{44}$$

Also, let

$$\frac{1}{\delta S}\int_{\delta A_{\alpha g}^b} \mathbf{q} \cdot \mathbf{n}^{\alpha g}\, da = \lambda^{\alpha g}(T^{b\alpha} - T^{bg}) \qquad \alpha = f, s \tag{45}$$

Substitution of (31) and (32) for $\mathbf{t}^f$ and $\mathbf{t}^s$ and of (44) and (45) into (39) and (40) in light of constraints (36) and (41) yields

$$[\varepsilon s^f \rho^f(E^f + \tfrac{1}{2}\mathbf{v}^f \cdot \mathbf{v}^f)(\mathbf{v}^f - \mathbf{W}) + \varepsilon s^f p^f(\mathbf{v}^f - \mathbf{W}) - \mathbf{q}^f] \cdot \mathbf{N}$$
$$= -\lambda^{fs}(T^{bf} - T^{bs}) - \lambda^{fg}(T^{bf} - T^{bg}) \tag{46}$$

$$[(1 - \varepsilon)\rho^s(E^s + \tfrac{1}{2}\mathbf{v}^s \cdot \mathbf{v}^s)(\mathbf{v}^s - \mathbf{W})$$
$$+ (1 - \varepsilon)p^f\mathbf{v}^s + \varepsilon s^f p^f\mathbf{W} - {}_e\mathbf{t}^s \cdot \mathbf{v}^s - \mathbf{q}^s] \cdot \mathbf{N}$$
$$= -\lambda^{fs}(T^{bs} - T^{bf}) - \lambda^{sg}(T^{bs} - T^{bg}) \tag{47}$$

Commonly in porous media flow, the contribution of mechanical and kinetic energies to heat transport is negligible and one may neglect the terms involving velocity squared, pressure, or stress. In this case, (46) and (47) reduce to

$$[\varepsilon s^f \rho^f E^f(\mathbf{v}^f - \mathbf{W}) - \mathbf{q}^f] \cdot \mathbf{N} = \lambda^{fs}(T^{bs} - T^{bf}) + \lambda^{fg}(T^{bg} - T^{bf}) \tag{48}$$

$$[(1-\varepsilon)\rho^s E^s(\mathbf{v}^s - \mathbf{W}) - \mathbf{q}^s] \cdot \mathbf{N} = \lambda^{fs}(T^{bf} - T^{bs}) + \lambda^{sg}(T^{bg} - T^{bs}) \tag{49}$$

If the α phase on both sides of the interface is intrinsically the same, E_1^α is equal to E_2^α by conditions (9). Then application of (18) in the absence of phase change eliminates the terms in (48) and (49) involving internal energy and these equations reduce to

$$[q^f] \cdot \mathbf{N} = \lambda^{fs}(T^f - T^s) + \lambda^{fg}(T^f - T^g) \tag{50}$$

$$[q^s] \cdot \mathbf{N} = \lambda^{fs}(T^s - T^f) + \lambda^{sg}(T^s - T^g) \tag{51}$$

where the thermal equilibrium condition (9*b*) has been employed (i.e., $T_1^\alpha = T_2^\alpha = T^{b\alpha}$ at the interface). These equations can be regarded as rather general interface conditions for situations where the fluid and solid attain different temperatures at each point. This may, for example, happen in geothermal reservoirs.

An important case of interest for ordinary groundwater problems is when the fluid, gas, and solid phases are in the state of local thermal equilibrium such that at all points within the medium, all phases have the same temperature. In such cases only one energy equation is required to describe the heat transport across the interface. Summation of (50) and (51) along with the corresponding condition that arises in a full treatment of the gas phase yields the desired relation

$$[q^m] \cdot \mathbf{N} = [\mathbf{q}^f + \mathbf{q}^s + \mathbf{q}^g] \cdot \mathbf{N} = 0 \tag{52}$$

where q^m is the total heat flux across the interface.

INTERFACE CONDITIONS FOR MULTIPHASE FLOW IN A POROUS MEDIUM

Consider a porous medium which is saturated by two immiscible fluids. Situations may exist that in neighboring domains separated by a macroscopic interface, two different sets of multiphase fluids are present. An example of such an interface is a phreatic surface, where there is water and air on one side and only water on the other side. Another example occurs in the water flooding of oil reservoirs. In this case, as water displaces oil, a transition zone is formed. Behind the transition zone the medium is filled with oil and water, whereas in front of it only oil exists. For both cases, in a macroscopic view, the transition zone may be modeled as a simple interface which moves with a velocity $\mathbf{W}$ as the water phase advances. Therefore the interface does not coincide with surfaces of discontinuity of the solid phase. The solid skeleton of the porous medium may be considered to be the same on both sides of the interface and thus all solid phase properties (such as density, permeability, porosity, etc.) are treated as continuous across the interface. Fluid phases may contain reacting components, some of which may be present in both fluids and which may be exchanged between the fluids across the interface. In the following, interface conditions are obtained for the case of oil/water. Superscripts $\alpha = o$ and $\alpha = w$ are used for the oil and water phases, respectively. Interface conditions for a phreatic surface are obtained analogously.

In general, balance equations (4)–(7) apply to each phase present in the system. In the problem under study, in front of the interface (say, side 2) $\varepsilon_2^w = 0$ and $\varepsilon_2^o = \varepsilon$, where ε is the porosity of the medium (and is continuous). Behind the interface (side 1) $\varepsilon_1^o = s_1^o\varepsilon$ and ε_1^w and $s_1^w\varepsilon$, where s^o and s^w are saturations with

$$s^o + s^w = 1 \tag{53}$$

Mass Balance

From (4), if the mass exchange between phases is small such that the integral on the right-hand side is negligible, the following mass equations are obtained:

$$[\varepsilon s^o \rho^o(\mathbf{v}^o - \mathbf{W})]_1 \cdot \mathbf{N}_1 = [\varepsilon \rho^o(\mathbf{v}^o - \mathbf{W})]_2 \cdot \mathbf{N}_1 \tag{54a}$$

$$[\varepsilon s^w \rho^w(\mathbf{v}^w - \mathbf{W})] \cdot \mathbf{N}_1 = 0 \tag{54b}$$

$$[(1-\varepsilon)\rho^s(\mathbf{v}^s - \mathbf{W})] \cdot \mathbf{N} = 0 \tag{54c}$$

Because solid phase properties are continuous, (54) reduce to

$$[\rho^o \mathbf{u}^o] \cdot \mathbf{N} = \varepsilon \mathbf{W}[s^o \rho^o] \cdot \mathbf{N} \tag{55a}$$

$$\mathbf{v}_1^w \cdot \mathbf{N}_1 = \mathbf{W} \cdot \mathbf{N}_1 \tag{55b}$$

$$[\mathbf{v}^s] \cdot \mathbf{N} = 0 \tag{55c}$$

Note that (54*b*) (or equation (55*a*)) indicates that the interface is a material surface for the water phase, i.e., no water crosses the interface. Equation (55*a*) relates the jump in mass flux of the oil to the amount of mass swept by the interface as it moves. Because ρ^o is a function of p^o and T^o and these will be shown to be continuous across the interface, ρ^o is also continuous and therefore (55*a*) reduces to

$$[\mathbf{u}^o] \cdot \mathbf{N} = \varepsilon \mathbf{W}[s^o] \cdot \mathbf{N} = -\varepsilon s_1^W \mathbf{W} \cdot \mathbf{N}_1 \tag{55d}$$

Mass Balance for Species

For species present in both fluid phases, (5) applies. If the species are nonadsorbing and in the light of (54*b*), interface conditions become

$$-[\mathbf{j}^{aw}] \cdot \mathbf{N} = -\frac{1}{\delta S}\int_{\delta A_{wo}^b} [\rho\omega^a(\mathbf{w} - \mathbf{v}) + \mathbf{j}^a] \cdot \mathbf{n}^{wo}\, da = R_{\text{het}}^{aw} \tag{56a}$$

$$[\varepsilon s^o \rho^o \omega^{ao}(\mathbf{v}^o - \mathbf{W}) - \mathbf{j}^{ao}] \cdot \mathbf{N} = -\frac{1}{\delta S}\int_{\delta A_{ow}^b} [\rho\omega^a(\mathbf{w} - \mathbf{v}) + \mathbf{j}^a] \cdot \mathbf{n}^{ow}\, da = -R_{\text{het}}^{aw} \tag{56b}$$

where the restriction (3) on interface transport is implicitly used.

Because the water phase is not present on side 2 of the interface, and assuming that R_{het}^{aw} is a known function $g(\omega_1^{aw} - \omega_1^{ao})$, (56) become

$$\mathbf{j}_1^{aw} \cdot \mathbf{N}_1 = g(\omega_1^{aw} - \omega_1^{ao}) \tag{57a}$$

$$\varepsilon s^o \rho^o(\mathbf{v}^o - \mathbf{W})[\omega^{ao}] \cdot \mathbf{N} - [\mathbf{j}^{ao}] \cdot \mathbf{N} = -g(\omega_1^{aw} - \omega_a^{ao}) \tag{57b}$$

where (54*b*) has been used. One needs to solve for ω^{aw} only in domain 1, but for ω^{ao} in both domains. Therefore (57*a*) is the sufficient condition for ω^{aw} but one more condition for ω^{ao} is required. That may be provided either by assuming an equilibrium relation between ω_1^{ao} and ω_2^{ao} in the form of condition (10) or by proposing a resistance-type equation for $\mathbf{j}_1^{ao} \cdot \mathbf{N}_1$ (or $\mathbf{j}_2^{ao} \cdot \mathbf{N}_2$) as a function of $(\omega_1^{ao} - \omega_2^{ao})$. Note that for species which are not exchanged between the fluid phases or are present only in one phase, the right-hand side of (57) will become identically zero.

Momentum Balance

Following a development similar to the one in the previous section, the momentum balance for each fluid phase reduces to

$$[p^\alpha] = 0 \qquad \alpha = o, w \tag{58}$$

which immediately implies

$$[p^o - p^w] = [p_c] = 0 \tag{59}$$

which p_c is the capillary pressure. This condition actually defines the location of the interface (see the section on Position of an Interface below).

Energy Balance

Assuming that the oil (or solid) phases on both sides of the interface are intrinsically the same, so that $E_1^\alpha = E_2^\alpha$ and conditions (19) hold, equations analogous to (50) and (51) will be obtained:

$$\mathbf{q}_1^w \cdot \mathbf{N}_1 = \lambda^{ws}(T_1^w - T_1^s) + \lambda^{wo}(T_1^w - T_1^o) \tag{60a}$$

$$[\mathbf{q}^s] \cdot \mathbf{N} = \lambda^{ws}(T_1^s - T_1^w) + \lambda^{so}(T_1^s - T_1^o) \tag{60b}$$

$$[\mathbf{q}^o] \cdot \mathbf{N} = \lambda^{so}(T_1^o - T_1^s) + \lambda^{wo}(T_1^o - T_1^w) \tag{60c}$$

These equations are supplemented by $T_1^s = T_2^s$ and $T_1^o = T_2^o$. If it is further assumed that all phases are in the state of local thermal equilibrium such that they all have a common temperature at a given point, (60) may be replaced by

$$[\mathbf{q}^m] \cdot \mathbf{N} = 0 \tag{61}$$

supplemented by the condition of continuity of temperature of the medium.

SALT WATER/FRESHWATER INTERFACE

Salt water and freshwater are actually two miscible fluids and normally the interface between them is not a sharp front. Nevertheless, as an approximation, and in a macroscopic view, often it is modeled as a simple interface between two immiscible fluids (see, for example, *Bear* [1979]). The interface may be moving and in general it does not coincide with surfaces of discontinuity in solid phase properties. Therefore similarly to the previous example, the solid skeleton properties are assumed to be continuous across the interface.

Mass Balance

Assume that the water and salt components of the fluid phase do not react with the solid phase and that reactions within the fluid phase are negligible. Therefore (4) with zero right-hand side applies

$$[\varepsilon^\alpha \rho^\alpha (\mathbf{v}^\alpha - \mathbf{W})] \cdot \mathbf{N} = 0 \tag{62}$$

Because salt and freshwater are assumed to behave as immiscible fluids, this equation applies to them separately. Thus because salt water (with $\alpha = w$) is present on only one side (e.g., side 1) and freshwater (with $\alpha = f$) is present only on the other side (side 2) of the interface, (62) reduces to

$$\varepsilon \rho_1^w (\mathbf{v}_1^w - \mathbf{W}) \cdot \mathbf{N}_1 = \varepsilon \rho_2^f (\mathbf{v}_2^f - \mathbf{W}) \cdot \mathbf{N}_2 = 0 \tag{63}$$

for salt water and freshwater. This equation indicates that the interface is a material boundary for salt water and freshwater. For the solid phase, recalling that ε and ρ^s are continuous, (62) yields

$$[\mathbf{v}^s] \cdot \mathbf{N} = 0 \tag{64}$$

Combination of (63) and (64) yields

$$\mathbf{u}_1^w \cdot \mathbf{N}_1 = \mathbf{u}_2^f \cdot \mathbf{N}_1 = \varepsilon(\mathbf{W} - \mathbf{v}^s) \cdot \mathbf{N}_1 \tag{65}$$

Note that the mass density of fluid phases may be discontinuous across the interface and no relationship between them will be obtained here. The mass densities are determined from equations of state of each fluid as a function of pressure, temperature, and salt concentration.

Species Mass Balance

For species a present in both salt and freshwater, (5) applies. However, the salt water phase exists only on side 1 and the freshwater only on side 2. Thus in light of (63), the interface conditions are

$$\mathbf{j}_1^{aw} \cdot \mathbf{N}_1 = \frac{1}{\delta S} \sum_{\beta \neq w} \int_{\delta A_{w\beta}^b} [\rho \omega^a (\mathbf{w} - \mathbf{v}) + \mathbf{j}^a] \cdot \mathbf{n}^{w\beta} \, da \tag{66a}$$

$$\mathbf{j}_2^{af} \cdot \mathbf{N}_2 = \frac{1}{\delta S} \sum_{\beta \neq f} \int_{\delta A_{f\beta}^b} [\rho \omega^a (\mathbf{w} - \mathbf{v}) + \mathbf{j}^a] \cdot \mathbf{n}^{f\beta} \, da \tag{66b}$$

where the term on the right side accounts for the adsorption of species on the solid phase and transfer between the fluid phases within the interface zone. To complete the set of required boundary conditions, (66) has to be supplemented with the equilibrium condition $\omega_1^w = f(\omega_2^f)$. If no solid phase adsorption is occurring, addition of (66*a*) and (66*b*) yields

$$\mathbf{j}_1^{aw} \cdot \mathbf{N}_1 = \mathbf{j}_2^{af} \cdot \mathbf{N}_2 \tag{67}$$

For species which are also unable to disperse from one fluid phase to the other, the fluxes in this equation will both be zero and the interface is a material surface for these species.

Momentum and Energy Balance

Equation (6) together with (63) and constitutive relations (31)–(33) yields

$$p_1^w = p_2^f \tag{68}$$

which is identical to the condition of local mechanical equilibrium at the interface. This equation is commonly employed at a salt water/freshwater interface. Equations (65) and (68) provide a complete set of interface conditions for flow problems. An additional equation is required to define the position of the interface; that is discussed in the next section.

For the energy balance, from (7) and using constitutive equations (31)–(33) and (43), equations similar to (46) and (47) are obtained. Then, in light of (63), (64), and (68), these equations may be approximated by

$$[\mathbf{q}^f] \cdot \mathbf{N} = \lambda^{fs}(T_1^f - T_1^s) \tag{69a}$$

$$[\mathbf{q}^s] \cdot \mathbf{N} = \lambda^{fs}(T_1^s - T_1^f) \tag{69b}$$

where it is assumed that local thermal equilibrium exists at the interface such that $T^{b\alpha} = T_1^\alpha = T_2^\alpha$. For the case that all phases have common temperature at a given point, (61) applies.

Position of a Moving Interface

The concept of a sharp interface between two fluids has been employed here to model the transition zone at a phreatic surface, at a salt water/freshwater interface, and at a water flooding front. In the mass balance equations (55) and (65), the normal component of the velocity $\mathbf{W} \cdot \mathbf{N}$ is still unknown. Therefore in this, and all moving boundary problems, one additional relation for the location (or velocity) of the interface is needed.

An interface can be described in space-time coordinates by an equation for a surface of the form

$$F(\mathbf{x}, t) = 0 \tag{70}$$

such that

$$DF/Dt = \partial F/\partial t + \mathbf{W} \cdot \nabla F = 0 \tag{71}$$

Equation (71) mathematically states that a particle moving with the interface remains on the interface. The gradient of the surface equation ∇F, is normal to the surface and therefore with $\mathbf{N} = \nabla F/|\nabla F|$, (71) becomes

$$DF/Dt = \partial F/\partial t + (\mathbf{W} \cdot \mathbf{N})|\nabla F| = 0 \tag{72a}$$

or

$$\mathbf{W} \cdot \mathbf{N} = \frac{\partial F/\partial t}{|\nabla F|} \tag{72b}$$

where $|\nabla F|$ denotes the magnitude of the vector ∇F.

This is a kinematic condition which supplements the flow conditions obtained earlier. However, one still needs to provide an appropriate expression for F. The choice of that expression depends on the nature of the interface and the type of process being considered. For example, for the case of a macroscopic oil-oil/water interface, $F = [p_c]$ (based on equation (59)); for a phreatic surface, $F = p - p_a$ (where p_a is the air pressure); and for a salt water-freshwater interface, $F = p^w - p^f$ (based on equation (68)).

Another example of a moving boundary is the case of phase change in a porous medium where, for example, a water phase may change to steam or ice. In that case, if T_0 is the temperature at which phase change occurs, the appropriate expression for F is

Could have T, T_0 const. for all p. [illegible]

$$F = T - T_0 \tag{73}$$

External Boundaries

The general conditions given in (4)–(7) can also be applied to situations where one side of the interface is not a porous medium. Examples include a porous formation adjacent to an impervious rock or a body of water. The first case is relatively simple when the rock is not soluble because advective fluxes are zero in the rock formation. The case of a soluble rock, for example a salt dome, has recently been studied by *Hassanizadeh and Leijnse* [1988]. The case of a surface water body which bounds a porous formation is treated below in detail.

Although the interface between a porous medium and a body of water may actually be considered as an external surface of the porous medium, sometimes the thermodynamics of the two systems are interrelated. For development of the jump conditions, the water body is effectively a medium with $\varepsilon = 1$. Note that the superscript w is used to denote the properties of the body of water and f is reserved for the water phase of the porous medium. Side one is considered to be the porous medium and side two to be the water reservoir.

Mass Balance

For the fluid and solid phases, with no phase change occurring, the interface equation (4) yields

$$\varepsilon\rho^f(\mathbf{v}^f - \mathbf{W}) \cdot \mathbf{N}_1 = \rho^w(\mathbf{v}^w - \mathbf{W}) \cdot \mathbf{N}_1 \tag{74}$$

$$(\mathbf{v}^s - \mathbf{W})|_1 \cdot \mathbf{N}_1 = 0 \tag{75}$$

With the Darcy velocity in the porous medium given by $\mathbf{u}^f = \varepsilon(\mathbf{v}^f - \mathbf{v}^s)$, (74) and (75) may be combined to obtain

$$\rho^f\mathbf{u}^f \cdot \mathbf{N}_1 = \rho^w(\mathbf{v}^w - \mathbf{W}) \cdot \mathbf{N}_1 \tag{76}$$

When the interface is stationary (i.e., $\mathbf{W} = 0$) and if $\rho^f = \rho^w$ then (76) simplifies to a statement of equality of volumetric fluxes:

$$\mathbf{u}^f \cdot \mathbf{N}_1 = \mathbf{v}^w \cdot N_1 \tag{77}$$

Mass Balance for Species

For the porous medium/water body interface when interphase transport is negligible, (5) becomes

$$[\varepsilon\rho^f\omega^{af}(\mathbf{v}^f - \mathbf{W}) + \mathbf{J}^{af}]|_1 \cdot \mathbf{N}_1 = [\rho^w\omega^{aw}(\mathbf{v}^w - \mathbf{W})]|_2 \cdot \mathbf{N}_1 \tag{78}$$

where the diffusive mass flux component in the water body has been neglected. By (76) and the definition of the Darcy velocity, (78) may be rewritten as

$$\rho^f(\omega^{af} - \omega^{aw})\mathbf{u}^f \cdot \mathbf{N}_1 + \mathbf{J}^{af} \cdot \mathbf{N}_1 = 0 \tag{79}$$

This relation, with $\mathbf{J}^{af}$ approximated using a Fickian relation, is the third-type boundary condition commonly employed at the boundary of a porous medium with a well-mixed reservoir [e.g., *Bear*, 1979, p. 249] where it is assumed that ω^{aw} is known. The form of this relation is unchanged when $\mathbf{W} \cdot \mathbf{N}_1 \neq 0$.

Momentum Balance

For this case, with negligible interphase transport in the interface region, equation (6) for the fluid and solid phases takes the forms

$$(\varepsilon\rho^f\mathbf{v}^f(\mathbf{v}^f - \mathbf{W}) - \mathbf{t}^f) \cdot \mathbf{N}_1 - (\rho^w\mathbf{v}^w(\mathbf{v}^w - \mathbf{W}) - \mathbf{t}^w) \cdot \mathbf{N}_1 = -\frac{1}{\delta S}\int_{\delta A_{fs}^b} \mathbf{t} \cdot \mathbf{n}^{fs}\, da \tag{80}$$

$$((1-\varepsilon)\rho^s\mathbf{v}^s(\mathbf{v}^s-\mathbf{W})-\mathbf{t}^s)|_1 \cdot \mathbf{N}_1 = \frac{1}{\delta S}\int_{\delta A^b_{fs}} \mathbf{t}\cdot\mathbf{n}^{fs}\,da \quad (81)$$

respectively.

Equation (80) is a very general interface condition for the fluid phase. Depending upon the actual flow situation, different assumptions may be applied to reduce it to a more specific form. For example, the following constitutive forms for t are appropriate if the water is assumed to be incompressible and Newtonian and the resistance of the interface zone to normal flow is neglected. The constitutive relation for the integral term is similar to (33):

$$\mathbf{t}^f = -\varepsilon p^f \mathbf{I} \quad (82)$$

$$\mathbf{t}^w = -p^w\mathbf{I} + \tfrac{1}{2}\mu^w(\nabla\mathbf{v}^w + (\nabla\mathbf{v}^w)^T) \quad (83)$$

$$\frac{1}{\delta S}\int_{\delta A^b_{fs}} \mathbf{t}\cdot\mathbf{n}^{fs}\,da = -(\varepsilon p^f - p^w)\mathbf{N}_1 + \varepsilon^b(p^f-p^w)\mathbf{N}_1 \quad (84)$$

Substitution of these forms into (80) yields

$$[\varepsilon\rho^f\mathbf{v}^f(\mathbf{v}^f-\mathbf{w})]\cdot\mathbf{N}_1 - [\rho^w\mathbf{v}^w(\mathbf{v}^w-\mathbf{W})]\cdot\mathbf{N}_1 + \tfrac{1}{2}\mu^w(\mathbf{N}_1\cdot\nabla\mathbf{v}^w + \nabla\mathbf{v}^w\cdot\mathbf{N}_1) = -\varepsilon^b(p^f-p^w) \quad (85)$$

If the flow is slow and because the interface is very narrow, the equilibrium condition $p^f = p^w = p^b$ is admissible. Then, using (74) and (76), (85) reduces to

$$\rho^f(\mathbf{v}^f-\mathbf{v}^w)\mathbf{u}^f\cdot\mathbf{N}_1 + \tfrac{1}{2}\mu^w(\mathbf{N}_1\cdot\nabla\mathbf{v}^w + \nabla\mathbf{v}^w\cdot\mathbf{N}_1) = 0 \quad (86)$$

The inner product of (86) with the unit vector tangent to the interface $\boldsymbol{\lambda}$ becomes

$$\rho^f\mathbf{u}^f\cdot\mathbf{N}_1(\mathbf{v}^f-\mathbf{v}^w)\cdot\boldsymbol{\lambda} + \frac{1}{2}\mu^w\left[\frac{\partial(\mathbf{v}^w\cdot\boldsymbol{\lambda})}{\partial N_1} + \frac{\partial(\mathbf{v}^w\cdot\mathbf{N}_1)}{\partial\lambda}\right] = 0 \quad (87)$$

When $\mathbf{W} = 0$, (77) may be applied to eliminate $\mathbf{v}^w\cdot\mathbf{N}_1$ and rewrite (87) as

$$\mathbf{v}^w\cdot\boldsymbol{\lambda} = \mathbf{u}^f\cdot\boldsymbol{\lambda} + \frac{\mu^w}{2\rho^f\mathbf{u}^f\cdot\mathbf{N}_1}\left[\frac{\partial(\mathbf{v}^w\cdot\boldsymbol{\lambda})}{\partial N_1} + \frac{\partial\mathbf{u}^f\cdot\mathbf{N}_1}{\partial\lambda}\right] \quad (88)$$

This equation is a third-type boundary condition for $(\mathbf{v}^w\cdot\boldsymbol{\lambda})$ and may be regarded as a generalization of the condition suggested by *Beavers and Joseph* [1967] and *Saffman* [1971].

For the solid phase interface condition, summation of (80) and (81), with (32) used as a constitutive representation of $\mathbf{t}^s$, and manipulations involving the use of (76) and (86), lead to the well-known condition

$$_e\mathbf{t}^s\cdot\mathbf{N}_1 = 0 \quad (89)$$

Energy Balance

The energy balance derivation for this case is analogous to that given in (39)–(51) for two adjacent porous media. The primary variation is that for the current case, one of the main adjacent domains has a porosity of 1. The most simplified result is given by (52)

$$(\mathbf{q}^f+\mathbf{q}^s)\cdot\mathbf{N} = \mathbf{q}^w\cdot\mathbf{N} \quad (90)$$

and applies when local equilibrium exists between the fluid and solid phases of the porous medium and the interface is stationary.

CONCLUSION

Simple interfaces are defined as the locus of a macroscopic surface of discontinuity in solid and/or fluid properties. They are narrow transition zones between different porous media or fluids with thickness on the order of magnitude of an averaging volume length scale. The transport within a simple interface may be neglected. Typically, one may assume that the media on the two sides of the interface are at a state of thermodynamic equilibrium defined by (9) and (10). The macroscopic equations of balance at a simple interface take the form of (4)–(7). The application of these equations is illustrated by employing them for obtaining appropriate interface conditions for flow, mass transport, and heat problems in a number of cases of practical interest.

For the case of two different porous media saturated with the same fluid, continuity of normal mass flux and pressure (equations (18) and (36)) serve as conditions for flow problems. Interface conditions for mass transport are given by (25) and (10) (or $\omega_1^{af} = \omega_2^{af}$), and for heat transport by (50)–(51) and (9*b*). These conditions are also applicable to unsaturated flow.

As examples of macroscopic discontinuity in multiphase flow, phreatic surface and a water flooding front in an oil reservoir are considered. For these cases, it is shown that the fluid pressures and capillary pressure are continuous across the interface. This condition together with (55) provide appropriate interface conditions for flow problems. For mass transport, (57) together with $\omega_2^{ao} = f(\omega_1^{ao})$ or $\mathbf{j}_1^{ao}\cdot\mathbf{N}_1 = f(\omega_1^{ao} - \omega_2^{ao})$, and for heat transport, (60) (or equation (61)) together with $T_1^\alpha = T_2^\alpha$ are appropriate.

The transition zone between freshwater and salt water is also treated as a simple interface. It is shown that the interface acts as a material surface and that the fluid pressure will be continuous across the interface. These two conditions are sufficient for flow problems. For mass transport, (66) together with $\omega_1^f = f(\omega_2^f)$, and for heat transport, (69) together with $T_1^\alpha = T_2^\alpha$ are the appropriate interface conditions.

Finally, the case of external boundaries is considered and the interface between a body of water and a porous medium is discussed as an example. It is shown that in addition to the continuity of normal flow velocity and pressure at the interface, a dynamic relationship exists between the tangential components of flow in the body of water and inside the porous medium. This condition may serve as an alternative to the no-slip boundary condition for a body of water in dynamic contact with a porous medium.

The development presented here provides profound insight to the nature of approximations behind a number of commonly employed interface and boundary conditions and it lays the foundation for obtaining correct interface condition for more complicated situations.

NOTATION

- b thickness of the interface zone between two media.
- da infinitesimal element of area.
- D characteristic size of the representative element of volume.

$\mathbf{D}^a$ dispersion tensor for species a.
E internal energy per unit mass.
f external supply in the general balance equation (2).
f_b^α contribution of interface to thermodynamic properties of α phase in (1).
F equation of a surface in space, in (70).
$\mathbf{g}$ gravity vector.
$\mathbf{i}$ nonadvective flux vector.
$\mathbf{I}$ unit tensor in three dimensions.
$\mathbf{j}$ dispersion flux vector.
$\mathbf{n}^{\alpha\beta}$ normal unit vector to a microscopic $\alpha\beta$ interface.
$\mathbf{N}$ normal unit vector in the macroscopic domain.
$\mathbf{N}_1(\mathbf{N}_2)$ normal unit vector to the interface at side 1 (or 2) pointing out of domain 1 (or 2).
p fluid pressure.
p_c capillary pressure.
$\mathbf{q}$ heat flux vector.
$R_{\text{het}}^{b\alpha}$ rate of species mass exchange as a result of heterogeneous reaction between phase α and all other phases in the interfaces.
s^α saturation of phase α.
t time.
$\mathbf{t}$ stress tensor.
${}_e\mathbf{t}^s$ effective stress tensor of solid phase.
T temperature.
$\mathbf{u}^\alpha$ Darcy velocity of phase α defined in (14).
$\mathbf{u}$ displacement vector of the solid phase.
$\mathbf{v}$ velocity vector.
$\mathbf{w}$ velocity of microscopic interfaces.
$\mathbf{W}$ velocity of macroscopic interfaces.

Greek Letters

$\delta A_{\alpha\beta}^b$ union of all $\alpha\beta$ interfaces within the interface zone.
δS cross-sectional area of the averaging volume.
ε porosity of the porous medium (also ε^b).
ε^α volume fraction of phase α (also $\varepsilon^{b\alpha}$).
$\lambda^{\alpha\beta}$ coefficient of local heat transfer between phases α and β.
$\boldsymbol{\lambda}$ tangent unit vector to the interface.
μ^α chemical potential of phase α.
μ^w viscosity of water.
ρ mass density.
ψ typical thermodynamic quantity.
ω mass fraction of species.

Special Notation

$-\alpha$ mass average of an α phase property of main domains.
$-b\alpha$ mass average of an α phase property of the interface.
$\langle\ \rangle_\alpha$ volume average of an α phase property of main domains.
$\langle\ \rangle_\alpha^b$ volume average of an α phase property of the interface.
$\sim b\alpha$ deviation of a property from its averaged value defined in (29).
∇ gradient operator in three dimensions.
∇' gradient operator in two dimensions (in the directions tangential to the interface).
[ ] jump of a property defined in (8).

$\sum_{\beta\neq\alpha}$ summation over all phases except phase α.

Subscripts

1 (or 2) thermodynamic property evaluated at side 1 (or 2) of the interface.

Superscripts

b properties associated with the interface.
s properties associated with the solid phase of a porous medium.
f properties associated with the fluid phase of a porous medium.
g properties associated with the gas phase.
o properties associated with the oil phase.
T transpose of a tensor.
w properties associated with the water phase.
α properties associated with the phase α.
$\alpha\beta$ properties associated with the microscopic α-β interfaces.

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