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Macroscale continuum mechanics for multiphase porous-media flow including phases, interfaces, common lines and common points

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This paper provides the tools needed for analysis of multiphase flow in porous media. Contributions are in four areas. First, theorems are provided that allow global scale integral equations to be localized at the porous medium scale. This is a more general approach than the traditional averaging of microscale point equations. Second, conservation equations for mass, momentum, energy and entropy for phases, interfaces, common lines, and common points are obtained. The inclusion of common lines and common points completes the full description of multiphase flow in porous media. Third, the entropy inequality is developed for the full multiphase system. The interaction terms between phases, interfaces, common lines, and common points provide a clear direction as to whether the entropy equation for each of these components may be used in the development of a constitutive theory or if the constitutive theory will depend on a combined entropy inequality statement. Fourth, the simplification of the system of equations is presented for the case of massless interfaces and common lines where these constituents are still capable of sustaining stress and containing energy. These latter forms are particularly useful in consideration of capillary pressure terms when the mass of the interface may be considered negligible. © 1997 Elsevier Science Limited. All rights reserved.

Key words: multiphase flow, porous media, averaging theory.

1 INTRODUCTION

During the past few decades significant progress has been made in developing general theories describing thermodynamic processes in general multiphase systems and in porous media. The primary aim of these derivations has been to obtain equations valid at a useful length scale of observation. Commonly, this scale is much larger than the scale of the minute structure and detailed occupation of space within the multiphase system. Two different approaches have been most often employed in these studies: mixture theories of continuum mechanics and averaging methods. The basic premises of these approaches have been to employ systematic procedures while retaining rigor such that arbitrariness and empiricism are avoided in the developments. Moreover, it has been the goal of these theories to avoid unwarranted simplifications and to include major features of the often complex multiphase systems. Thus, for example, multiphase theories have been developed that account for the presence of interfacial regions separating various phases at the microscale.

Early theories, however, considered these interfaces to be singular surfaces devoid of thermodynamic quantities. In these older approaches, the interfaces were simply modeled as surfaces of discontinuity in phase properties unable to affect thermodynamic processes on their own (e.g. in Refs 2, 3, 8, 15, 16, 27). Subsequent theories have assigned thermodynamic properties to the interfaces and thus include the effects of interfaces on the medium behavior at the macroscale (e.g. 9, 17, 20). These newer theories have been employed to derive basic equations describing two-phase flow processes and flow in unsaturated media[10,11,18,19] and flow in clayey materials.[1] The results of these studies have shown that inclusion of interfacial properties is essential if gross errors in the description of some multiphase systems are to be avoided. Based on these advances, it has become