Waves and Wave-Driven Processes in the Nearshore: Part 1

This lecture

• different wave types
• wave generation
• wave characteristics (anatomy)
• deep-water waves
• from deep to intermediate water

Why study waves?
Why study waves?

• transfer of energy (wind → breaking)
• generate other type of waves and currents
• determine morphology in the nearshore
  • erosion
  • sedimentation

Wave Classification

• period (or, frequency)
  • gravity waves
  • infragravity waves
  • tides
• relationship with wind:
  • gravity waves: sea and swell
• structure
  • infragravity waves: edge and leaky
Wave Measurements

• In situ (in, on, below the waves)
  • buoys
  • pressure gages
  • surface gages
  • ...

• Remote sensing (above the waves)
  • visual / airplane / satellites
  • e.g., radar

buoys
pressure sensor
surface gage
Wave Anatomy (1)

- **Direction of wave advance**
- **Height**
- **Wavelength**
- **Still water level**
- **Crest**
- **Trough**

**Frequency (f):** Number of wave crests passing point A or point B each second

\[ f = \frac{1}{T} \]

**Period (T):** Time required for wave crest at point A to reach point B

Wave Anatomy (2)

- **Wave Record**
- **Height Statistics**
- **Spectral Analysis**
Wave Generation (1)

- initial formation: turbulent eddies in wind field
  - capillary waves
- subsequent growth: sheltering effect
  - pressure difference

Wave Generation (2)

- during growth:
  - short waves steepen and break
  - energy absorbed by longer waves
  - with time, more and more energy at larger periods
  - sweeping mechanism
- wind → capillary waves → larger-period waves

generation and growth of wind waves is a function of
(1) wind velocity
(2) wind duration
(3) fetch
Deep water, fully developed sea: prediction of H and T given U, F and t

Example: U = 15 m/s, F = 200 km, duration t = 6 hours

2.9 m

5.8 s
Example: $U = 15 \text{ m/s}$, $F = 200 \text{ km}$, duration $t = 18 \text{ hours}$

Wave generation revisited:
- wind $\rightarrow$ capillary waves $\rightarrow$
- longer-period waves

Wave generation at Sylt (D)
Offshore winds - JONSWAP

Transformation at sea
- Dispersion
  - Waves of different length travel at different speeds
- Wave interference
  - wave groups
Dispersion

- DISPERSION RELATIONSHIP:

\[ \sigma^2 = gk \tanh(kh) \]

\[ \sigma = \frac{2\pi}{T} \]

\[ k = \frac{2\pi}{L} \]

- Multiplying by \( L \) and reducing we can isolate the wavelength:

\[ L = \frac{8T^2}{2\pi} \tanh \left( \frac{2\pi h}{L} \right) \]

- Wave celerity:

\[ c = \frac{xT}{2\pi} \tanh \left( \frac{2\pi h}{L} \right) \]

**Ah is the relative depth**

\[ \sigma^2 = gk \tanh(kh) \]

Large \( Ah \)
Deep water \( \tanh(kh) = 1 \)
Shallow water \( \tanh \rightarrow kh \)

**TABLE 5.2 EQUATIONS DERIVED FROM LINEAR WAVE THEORY**

<table>
<thead>
<tr>
<th>Variable</th>
<th>General Expression</th>
<th>Deep Water</th>
<th>Shallow Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase velocity</td>
<td>( c = \frac{gk}{2\pi} \tanh \left( \frac{2\pi h}{L} \right) )</td>
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</tr>
<tr>
<td>Wave length</td>
<td>( L = \frac{8T^2}{\pi} \tanh \left( \frac{2\pi h}{L} \right) )</td>
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<tr>
<td>Horizontal kinetic energy</td>
<td>( E = \frac{1}{2} \rho u^2 )</td>
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</tr>
<tr>
<td>Vertical kinetic energy</td>
<td>( E = \frac{1}{2} \rho \omega^2 )</td>
<td>( E = \frac{1}{2} \rho \omega^2 )</td>
<td>( E = \frac{1}{2} \rho \omega^2 )</td>
</tr>
</tbody>
</table>

Home exercise: \( h = 15 \text{ m}, T = 10 \text{ s}, L = ? \)
Dispersion

- Waves of different length travel at different speeds.
- In deep water the wavelength (\(L\)) and wave celerity (\(C\)) are proportional to the wave period (\(T\)):\
  \[
  L = \frac{gT^2}{2\pi} \quad \text{and} \quad C = \frac{gT}{2\pi}
  \]

These equations are only valid for deep water!

Sea and Swell

- sea: many waves with different heights and period
- swell: waves escape from wind
  - waves with larger periods travel faster (\(c \propto T\))
  - waves are sorted out by period
  - dispersion of swell

- swell dispersion
Wave Interference

- Waves are not a single sinusoid
- Waves are often grouped (5 – 10 waves in a group)

Real waves are a summation of sinusoids
- Here, just 2; in practice, an awful lot

Wave Group Velocity

- For waves to propagate into undisturbed water at large depths then some energy must be dissipated in setting up the initial oscillation

Undisturbed Surface

5 4 3 2 1

6 5 4 3 2

Group Velocity (cG = n c):
- Deep water: ½ of the individual wave velocity (n = ½)
- Shallow water: approaches the velocity of the individual wave (n = 1)
Wave Energy

Wave energy ($E_{\text{wave}}$) = potential wave energy ($E_{\text{potential}}$) + kinetic wave energy ($E_{\text{kinetic}}$)

\[ E_{\text{wave}} = E_{\text{potential}} + E_{\text{kinetic}} \]

\[ E_{\text{wave}} = \frac{1}{8} \rho g H^2 \]

Energy flux ($P$) = wave energy ($E_{\text{wave}}$) * group velocity ($c_g$)

\[ P = E_{\text{wave}} \cdot c_g \]

Deep to intermediate water

Waves start to feel the sea bed!!

Changes:
- orbital motion becomes an ellips
- wave shape changes, from linear to non-linear theory
- mass transport becomes non-zero
- wave shoaling
- wave refraction
- wave diffraction

Orbital Motion

![Orbital Motion Diagram]

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Wave Shape changes:

Airy and Stokes Wave

Airy wave = simple sinusoid:

\[ \eta(x,t) = \frac{H}{2} \cos(kx - \omega t) \]

Stokes wave = simple sinusoid + disturbance:

\[ \eta = \frac{H}{2} \cos(kx - \omega t) + \frac{2H^3}{2L} \left( \frac{2 + \cosh(2kh)}{\sinh(kh)} \right) \cos(2(kx - \omega t)) \]

If \( H/L \rightarrow 0 \) the above equation reduces to the Airy solution.
You can think of a Stokes wave as an Airy wave with a smaller wave of 1/2 period added to it.

More time is spent under the trough than under the crest.

Equal time is spent under the trough as under the crest.

An important prediction of Stokes’ Theory is that:

Water particle orbits are **NOT CLOSED**.

Thus a MASS TRANSPORT or WAVE DRIFT is predicted:

\[
\sigma = \frac{1}{2} \left[ \frac{\text{d}H}{\text{d}t} \right] \frac{C}{\text{tanh}(2kL)} \frac{\text{sinh}[2k(\zeta + h)]}{\text{sinh}(4h)}
\]

Downwave Mass Transport (STOKES DRIFT)

Assumes an infinitely long length of wave travel and a non-viscous fluid.
• Longuet-Higgins (1953) subsequently provided a solution of a finite length of travel and for the presence of viscosity. Thus the flow must satisfy CONTINUITY and the equations suggest:

**DOWNWAVE (landward) MEAN flow**
at the water surface & bed

**UPWAVE (seaward) return flow at mid-depth**

Direction of Waves

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**From deep to intermediate depth**

What will happen to:      | Answers:
--- | ---
1. Wave period $T$?      | 1. Remains the same
2. Wave length $L$?      | 2. Reduces
3. Wave speed $c$?       | 3. Reduces
4. Group velocity $c_g$? | 4. Reduces
5. Wave energy flux?     | 5. Remains the same

---

**Shoaling**

Energy flux deep water = energy flux intermediate depth

$$E \cdot c_g \text{ (deep)} = E \cdot c_g \text{ (intermediate)}$$

Because of the conservation of energy flux, the wave height will increase when a wave field enters intermediate waters. This is called **shoaling**.
Refraction
Wave velocity decreases with decreasing water depth.
Waves travel slowly in shallower water (Snell's law).

**Refraction**

\[
\sin \alpha = \frac{c}{c_\infty} \sin \alpha_\infty
\]

- Assumptions:
  - Parallel depth contours
  - No energy losses during wave propagation
  - No lateral leaking of energy

Wave energy flux between the wave rays = constant from deep to shallow water.
Shoaling and Refraction

when the water depth decreases:

\[
\frac{H}{H_\infty} = \left[\frac{c_\infty}{2nc}\right]^{0.5} \left[\frac{s}{s_\infty}\right]^{0.5}
\]

 increases decreases

Complex Refraction Patterns
Complex Refraction Patterns

Diffraction

From deep to intermediate depth

Waves start to feel the shadow zones. Energy is leaking away. Energy losses reduce wave heights.

Changes:

- Orbital motion becomes an ellips
- Wave shape changes, from linear to non-linear theory
- Mass transport becomes non-zero
- Wave shoaling
- Wave refraction
- Wave diffraction

From deep to intermediate depth