Anisotropy of Earth’s inner core intrinsic attenuation from seismic normal mode models

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A B S T R A C T

The Earth’s inner core, the slowly growing sphere of solid iron alloy at the centre of our planet, is known to exhibit seismic anisotropy. Both normal mode and body wave studies have established that, when the global average is taken, compressional waves propagate faster in the North–South direction than in the equatorial plane. Recent body wave studies also indicate that this fast direction may be more attenuating, and interpret this anisotropic attenuation in terms of anisotropic scattering due to inner core texturing. Here we use the Earth’s normal modes to study the attenuation anisotropy of both compressional and shear waves in the inner core. As normal modes have wavelengths several orders of magnitude longer than estimates of inner core grain size, any attenuation anisotropy quantified using normal modes must reflect the anisotropy of intrinsic (viscoelastic) attenuation of the crystalline inner core alloy. By inverting zonal anelastic and elastic normal mode splitting function coefficients of twenty inner core sensitive modes, we construct models of inner core intrinsic attenuation and velocity anisotropy. We find that, for compressional waves, the North–South direction is both fast and more strongly attenuating. The existence of intrinsic inner core attenuation anisotropy can be interpreted in terms of anisotropic Zener relaxation in the metallic alloy comprising the inner core. Such anisotropic Zener relaxation has only been observed in the presence of solute atoms, and is thus entirely consistent with the presence of a few atomic per cent of light elements in the Earth’s inner core.

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1. Introduction

The Earth’s inner core, the centremost part of our planet, grows by slow solidification of the liquid outer core (Jacobs, 1953), a process which preferentially partitions light elements into the outer core liquid. Large-scale seismological studies, such as those conducted to build radial Earth models, most famously PREM (Dziewoński and Anderson, 1981), find that the density of the inner core is slightly lower than that of pure iron. This indicates that some light elements must necessarily be incorporated into the growing inner core (Jephcoat and Olson, 1987). Seismological studies using both normal modes and body waves show that compressional velocity in the inner core is anisotropic, with the North–South (polar) direction appearing faster than the East–West (equatorial) plane (e.g. Woodhouse et al., 1986; Morelli et al., 1986). Some recent body wave observations also suggest that the inner core exhibit anisotropy in compressional wave attenuation (Souriau and Romanowicz, 1996; Niu and Wen, 2002; Yu and Wen, 2006), with stronger attenuation in the fast polar direction. This attenuation anisotropy has previously been interpreted in terms of texturing of the inner core, which would give rise to direction-dependent scattering attenuation (Cormier and Li, 2002).

However, body waves, the only seismological data so far used to constrain inner core attenuation anisotropy, are unsuitable for distinguishing between scattering and intrinsic attenuation, as they are attenuated by both mechanisms. Moreover, body waves do not constrain attenuation or velocity anisotropy of shear waves in the inner core, as observations of the relevant phases are scarce (Deuss et al., 2000). On the other hand, normal modes — whole-Earth free oscillations at very low frequencies (<1 mHz) — have inner core wavelengths on the order of 1000 km laterally and 100 km radially, and are thus unaffected by scattering from smallscale heterogeneities for realistic estimates of inner core grain size, 50 m–20 km (Bergman, 1998; Calvet and Margerin, 2008; Monnereau et al., 2010). Normal modes then directly probe the intrinsic attenuation of the inner core.

Here we make use of normal mode splitting function coefficients to study the attenuation anisotropy of the inner core.
Previously, only the elastic splitting function coefficients — pertaining to velocity and density heterogeneities — were employed, with the assumption that the anelastic splitting function coefficients — pertaining to attenuation variations — would be too small to be of importance. However, we have recently extended the splitting function technique, finding that the anelastic splitting function coefficients are large enough to be measured for inner core sensitive modes; furthermore, including them in our forward modelling allows us to calculate synthetic seismograms that match observed data significantly better (Mäkinen and Deuss, 2013b). Similar to the elastic splitting function coefficients, the observed anelastic splitting function coefficients are dominated by zonal symmetry (i.e. independent of longitude), an observation which indicates the existence of cylindrical anisotropy in attenuation in the Earth’s inner core.

Here we shall use these normal mode observations to show that intrinsic attenuation in the inner core is anisotropic. We also demonstrate that anisotropic Zener relaxation in iron offers an explanation to the physical origin of this anisotropy of intrinsic attenuation (LeClaire and Lomé, 1954; Seraphim and Nowick, 1961; Berry, 1961). This mechanism requires that a few per cent of light elements, as solute, be present in the iron alloy inner core.

2. Theoretical background and methods

2.1. Normal modes and splitting function coefficients

Normal modes, the Earth’s low-frequency, long-wavelength free oscillations, are standing waves along the radius and surface of the planet. They comprise different geometric patterns of oscillation for the entire three-dimensional Earth, with nodes of no motion along the Earth’s radius and surface. Each normal mode has its own discrete characteristic oscillation frequency and quality factor; together these comprise the complex eigenfrequency ω of the mode. Normal mode frequencies are in the millihertz range, much lower than the typical inner core compressional wave frequency of 1 Hz for body waves.

Normal modes that are sensitive to the inner core belong to a class of modes called spheroidal modes, nSl. These comprise P–SV type wave motion, and are characterised by their radial order n and angular order l. Each spheroidal normal mode, or multiplet, comprises 2l + 1 singlets, labelled using the azimuthal order m. For a spherical, non-rotating, elastic and isotropic (SNRE) Earth, the 2l + 1 singlets of a given multiplet are degenerate, that is, they share the same complex eigenfrequency. Any deviation from a simple SNRE structure leads to splitting of normal modes — the 2l + 1 singlets are no longer degenerate. Rotation, ellipticity, velocity and attenuation anisotropy and heterogeneity split the singlets in complex frequency.

In this work we make use of the self-coupling approximation: we treat each normal mode as isolated, and only consider the interaction between the 2l + 1 singlets of that particular mode, neglecting all interactions between singlets of different modes. Under this approximation, the odd standing wave contributions cancel each other, and thus Earth structure of even degree only can be studied. Should some odd-degree structure be present, it would be averaged into its even-degree counterpart, for example degree three (hemispherical) variations into degree two patterns. Thus distinguishing between actual even and odd-degree contributions would not be possible.

Using the self-coupling approximation, the problem of modelling inner core structure is separated into two independent steps. Firstly, observed spectra are inverted, in a non-linear inversion, for a set of coefficients, called the splitting function coefficients (Giardini et al., 1988), σst. These splitting function coefficients σst are divided into elastic splitting function coefficients cst and anelastic splitting function coefficients dst:

\[ σ_{st} = c_{st} + id_{st} \]  (1)

We have recently made the first extended dst measurements and shown that these coefficients are important for normal modes with inner core sensitivity (Mäkinen and Deuss, 2013b). Our focus in the present work is the second step of the scheme: after the splitting function coefficients have been measured, they are inverted for Earth structure. The coefficients are linearly dependent on Earth structure of degree s and order t; we shall consider the inversion of splitting function coefficients for inner core velocity and attenuation anisotropy.

2.2. Velocity and attenuation anisotropy from splitting function coefficients

The globally averaged inner core velocity anisotropy is usually considered to be cylindrically symmetric, with the North–South axis as the symmetry axis; this is a type of transverse isotropy, as all directions perpendicular to the symmetry axis are equivalent (that is, there are no longitudinal variations). Such cylindrical models are parametrized (Woodhouse et al., 1986; Tromp, 1993, 1995) in terms of three depth-dependent parameters αR, βR and γR, which relate to the (real) Love parameters AR, CnR, LR and FR (Love, 1927). When cylindrically symmetric attenuation anisotropy is considered in addition to velocity anisotropy, it can be shown (Mäkinen and Deuss, 2013a) that both the Love parameters and the anisotropy parameters α, β and γ become complex; the anisotropy parameters are then given by

\[ α^R = \frac{CR + A_R^R}{A_0^R}, \quad α^I = \frac{CI - A_I^I}{A_0^I} \]  (2)
\[ β^R = \frac{LR - N_R^R}{A_0^R}, \quad β^I = \frac{LI - N_I^I}{A_0^I} \]  (3)
\[ γ^R = \frac{A_R^R - 2N_R^R - F_R}{A_0^R}, \quad γ^I = \frac{A_I^I - 2N_I^I - F_I}{A_0^I} \]  (4)

where the reference value, defined at the centre of the spherically symmetric reference model PREM (Dziewo´nski and Anderson, 1981), is A0R = ρ0v02R0 for density ρ and compressional wave velocity vp. The superscripts R and I now denote the real and imaginary parts of each of the anisotropy parameters, respectively. αR and βR describe the attenuation anisotropy; αR and γR give the velocity anisotropy. αI describes the compressional wave attenuation anisotropy of the inner core, with a positive value of αI indicating North–South (polar) direction being more attenuating than directions in the East–West (equatorial) plane; αR describes the compressional wave velocity anisotropy, with a positive value of αR indicating a faster polar direction. In an equivalent manner, βI and βR describe the attenuation and velocity anisotropy, respectively, of shear waves. Both α and β relate to the difference between the polar symmetry direction and directions that lie in the equatorial plane perpendicular to this direction; the attenuation and velocity anisotropy of compressional and shear waves propagating at intermediate angles is described by the parameters γI and γR, respectively.

Using AR, βR, γR, αR, βI and γI, the anisotropy of P-wave velocity vp and attenuation qvp, equatorially polarized S-wave velocity vs, and attenuation qvs, are given (Mäkinen and Deuss, 2013a)

\[ \frac{δv_p}{v_p} = \frac{1}{2} \frac{A_0^R}{A_R^R} (4β_R^2 - 2γ_R^R) \cos^2 ξ \]
\[
\delta q^\beta = \frac{\dot{A}_R^2}{A_R^2} (4\beta^l - 2\gamma^l) \cos^2 \xi + \frac{\dot{A}_R^2}{A_R^2} (\alpha^l - 4\beta^l + 2\gamma^l) \cos^4 \xi
\]

(5)

\[
\delta \nu_{\text{true}} = \frac{\dot{A}_R^2}{2 A_R^2} (\alpha^l - 4\beta^l + 2\gamma^l) \cos^4 \xi
\]

(6)

\[
\delta q^s_{\text{true}} = \frac{\dot{A}_R^2}{2 A_R^2} (\alpha^l - 4\beta^l + 2\gamma^l) \cos^2 \xi - \cos^4 \xi
\]

(7)

where \( \xi \) is the angle between the ray and the polar symmetry axis, taken to be the Earth's rotation axis, and attenuation \( q \) relates to quality factor \( Q \) by \( q = 1/Q \).

The overall contribution of non-hydrostatic elastic lateral heterogeneity anywhere in the Earth to all the elastic splitting function coefficients \( c_{st} \) (not just the \( c_{st} \) inverted in this work) can be written

\[
c_{st} = \int_0^a \delta m_{st}(r) \cdot K_s(r) dr + \sum_d \delta h^d_{st} H^d_s
\]

(11)

where the coefficients \( \delta m_{st} \) are the harmonic coefficients of Earth elastic heterogeneity, comprising sensitivity to the incompressibility (bulk modulus) \( \kappa \), the rigidity (shear modulus) \( \mu \), and the density \( \rho \); \( \delta h^d_{st} \) are coefficients for discontinuity contribution; \( K_s(r) \) and \( H^d_s \) are known sensitivity kernels describing the responses to perturbations in these parameters (Woodhouse, 1980); and \( a \) is the radius of the Earth.

Likewise, the overall contribution of lateral variations in anelasticity anywhere in the Earth to all anelastic splitting function coefficients \( d_{st} \) is

\[
d_{st} = \frac{1}{2} \alpha_0^2 \int_0^a \left( k_0 \delta q_{kst} K^s_{kst} + \mu_0 \delta q_{\mu st} M^s_{\mu st} \right) r^2 dr
\]

(12)

Here, the coefficients \( \delta q_{kst} \) and \( \delta q_{\mu st} \) describe the lateral variation of the attenuation in incompressibility and rigidity of the Earth. \( K^s_{kst} \) and \( M^s_{\mu st} \) are again known sensitivity kernels (Woodhouse, 1980).

Reparametrizing the inner core velocity anisotropy part of Eq. (11), the inner core velocity anisotropy contribution to the zonal elastic splitting function coefficients \( c_{s0} \) \((s = 2, 4)\) is given by

\[
c_{s0}^{\text{Cam}} = \int_0^{R_C} \left[ \alpha^R(r) K_\alpha(r) + \beta^R(r) K_\beta(r) + \gamma^R(r) K_\gamma(r) \right] dr
\]

(13)

where \( R_C \) is the radius of the inner core, and \( K_\alpha, K_\beta \) and \( K_\gamma \) are depth-dependent sensitivity kernels, describing how a given mode reacts to perturbations in the anisotropy parameters. These kernels are constructed using the kernels described by Mochizuki (1986) in their Appendix B. Likewise, rewriting the inner core attenuation anisotropy part of Eq. (12), the inner core attenuation anisotropy contribution to the zonal anelastic splitting function coefficients \( d_{s0} \) \((s = 2, 4)\) is given by

\[
d_{s0}^{\text{Cam}} = \int_0^{R_C} \left( \alpha^l(r) K_\alpha(r) + \beta^l(r) K_\beta(r) + \gamma^l(r) K_\gamma(r) \right) dr
\]

(14)

where the kernels \( K_\alpha, K_\beta \) and \( K_\gamma \) are the same as those above. Eqs. (13) and (14) are used in inverting the measured splitting function coefficients \( c_{s0} \) and \( d_{s0} \) for velocity and attenuation anisotropy respectively.

The present theoretical approach does not use the non-zonal coefficients \((t \neq 0)\). Likewise, the \( s = 0 \) coefficients — which could be used to invert for radial variations of attenuation and velocity in the inner core and the rest of the Earth — are not used in the present work, predominantly due to the limited amount of inner core information that could be reliably extracted from such a small number of coefficients. In general, the \( c_{00} \) and \( d_{00} \) coefficients are non-zero for most normal modes, including modes that are not sensitive to the Earth's inner core. Therefore, in order to invert the \( c_{00} \) and \( d_{00} \) coefficients of inner core sensitive modes for inner core structure, one would first need to construct radial models of whole Earth velocity (\( c_{00} \)) and attenuation (\( d_{00} \)), and then subtract the crustal, mantle and possible outer core contributions from the inner core sensitive mode measurements as corrections. Only then could the radial structure of the inner core itself be quantified by inverting the corrected coefficients. This situation is very different from the other \( t = 0 \) coefficients, for which the contribution from structure outside the inner core is very small in comparison to the inner core contribution (see below). Therefore the models described below strictly relate to anisotropy of attenuation and velocity, with no new information, outside the reference model, relating to the absolute values of attenuation and velocity.

Eqs. (13) and (14) contain, respectively, the inner core velocity and attenuation anisotropy contributions to the splitting function coefficients. These contributions are equal to the measured splitting function coefficients assuming all contributions from volumetric perturbations in the crust, mantle and outer core, and from topographic perturbations to discontinuities at various points along the radius of the Earth, as well as non-anisotropy-related inner core contributions, are absent or negligibly small. Alternatively, the measured coefficients can be corrected for any such effects that are considered significant, after which the relationships in Eqs. (13) and (14) hold for the corrected coefficients; this has been our approach.

2.3 Inversion method

We use standard linear least-squares techniques (Tarantola and Valette, 1982a, 1982b) in inverting the elastic and anelastic splitting function coefficients for velocity and attenuation anisotropy. The inversions are damped, with the damping parameter ranging over ten orders of magnitude; the final results are found to converge with respect to damping. Both \( L_1 \) and \( L_2 \) misfits are considered when selecting the final damping parameter, with good agreement between all converged models. A wide range of different starting models, ranging from zero to complete dominance of any of the three anisotropy parameters, for both velocity and attenuation anisotropy are tested; the final results are found to be completely independent of the starting model used, and the results given have been obtained with zero starting models.

In our inversions, we choose a simple depth parametrization in terms of even-degree radial polynomials. Radial characteristics of the final attenuation (radially constant) and velocity \((a + b^2)\) radial anisotropy models are discussed in more detail below. We do note that as the sensitivity of normal modes to perturbations in structure, including anisotropy, decreases to zero towards the centre of the Earth, we would not expect to be able to resolve any structure between the radii of approximately 0–300 km. This leaves the centremost part of the Earth necessarily a null space in normal mode terms, and our velocity and attenuation anisotropy
models described below are not intended to characterise structure in this region.

2.4. Misfit and robustness

The misfit of individual velocity and attenuation anisotropy models to observed elastic and anelastic splitting function coefficients, respectively, is quantified using the variance reduction, VR. For the attenuation anisotropy model, the variance reduction is defined in terms of the misfit $L_2$, given by

$$L_2 = \frac{1}{N} \sum_{s=2}^{4} \sum_{i} \left( c_{\alpha,s}^{\text{obs},i} - c_{\alpha,s}^{\text{pred},i} \right)^2 \tag{15}$$

where $c_{\alpha,s}^{\text{obs},i}$ and $c_{\alpha,s}^{\text{pred},i}$ are the observed and predicted anelastic splitting function coefficients, respectively, of the $i$th mode, for $s = 2$ or $s = 4$, and $N$ is the total number of coefficients used in building the anisotropy model. Then the variance reduction is

$$\text{VR} = \frac{\langle (d_{\alpha})^2 \rangle - L_2}{\langle (d_{\alpha})^2 \rangle} \tag{16}$$

where $\langle (d_{\alpha})^2 \rangle$ denotes the average squared coefficient size, given by

$$\langle (d_{\alpha})^2 \rangle = \frac{1}{N} \sum_{s=2}^{4} \sum_{i} (c_{\alpha,s}^{\text{obs},i})^2 \tag{17}$$

The variance reduction of the velocity anisotropy model is calculated in an equivalent manner, using the elastic splitting function coefficients $c_{\alpha}$. Cross-validation methods (see e.g. Efron and Tibshirani, 1993) are used to obtain the errors for the attenuation and velocity anisotropy models. We quantify the model confidence by leaving out a tenth of the modes used, and then inverting the remaining coefficients again. This procedure is repeated ten times, with different modes omitted each time. In particular, in one of the cross-validation runs we leave out the mode $\nu_S^3$, whose extreme energy density in the inner core may overpredict the final models.

3. Data

3.1. Inner core sensitive modes

For modelling inner core attenuation anisotropy, we use the anelastic splitting function coefficients $d_{\alpha}$ and $d_{\alpha}$ we have measured (Mäkinen and Deuss, 2013b) for the inner core sensitive modes $2S_3, 3S_2, 8S_5, 9S_3, 9S_4, 11S_4, 11S_5, 13S_1, 11S_2, 13S_3, 13S_6, 15S_1, 15S_4, 16S_5, 18S_3, 18S_4, 20S_1, 21S_6, 23S_2, and 27S_2. These modes have been selected for inversion as their elastic and anelastic splitting function coefficients can be robustly measured under the self-coupling approximation. For most of these modes, this is due to the absence of other normal modes at nearby frequencies to which these inner core sensitive modes could couple through Earth structure of sufficiently low order. As for $S_3^5$ — which cross-couples to the inner core confined mode $S_3^{10}$ (Deuss et al., 2010) — and $16S_5$ — which cross-couples to the inner core confined mode $17S_4$ — our test measurements (Mäkinen, 2013) find that the strong coupling via odd-degree inner core structure has no effect on the even-degree anelastic splitting function coefficients of these modes themselves, and thus the use of the even-degree $d_{\alpha}$ measured under the self-coupling approximation is appropriate.

The depth dependent inner core anisotropy kernels $K_{x,y,z}$ of these 20 modes are shown in Fig. 1. We note that $\alpha^R$ and $\alpha^I$ are the dominant velocity and attenuation anisotropy parameters, respectively, in terms of mode sensitivity. It is therefore expected that they be the most stable parameters, which is indeed observed to be the case in our modelling. For modelling the velocity anisotropy, we use the corresponding elastic splitting function coefficients $c_{\alpha}$ and $c_{\alpha}$ we have measured (Mäkinen and Deuss, 2013b).

3.2. Ellipticity and mantle corrections

The inner core sensitive modes employed here are also sensitive to structure elsewhere in the Earth. This is compensated for by applying corrections due to both mantle structure and the Earth’s ellipticity of figure to both the elastic and the anelastic splitting function coefficients prior to inverting them for anisotropy models. Topographic perturbations to radial discontinuities in the Earth are not considered in the present study due to their expected small contributions. Contributions due to ellipticity arise for both the elastic $c_{\alpha}$ and the anelastic $d_{\alpha}$ coefficients; ellipticity does not contribute to $c_{\alpha}$ or $d_{\alpha}$. The elastic $c_{\alpha}$ are corrected prior to measurement (Mäkinen and Deuss, 2013b), and need not be corrected further; anelastic ellipticity corrections are applied to the measured $d_{\alpha}$ coefficients. The ellipticity corrections applied to the $d_{\alpha}$ coefficients are similar to the standard ellipticity corrections applied to $c_{\alpha}$ (see e.g. Dahlen and Tromp, 1998), except that density is left unperturbed. These ellipticity corrections applied to the $d_{\alpha}$ coefficients are small, of the order of 0.01 or 0.1 μHz, compared to typical $d_{\alpha}$ coefficient size of the order of 1 μHz. The anelastic ellipticity corrections are negative for all modes considered herein.

The elastic splitting function coefficients $c_{\alpha}$ are corrected for velocity and density structure in the whole mantle and crust, as described in the models S20RTS (Ritsma et al., 1999) and CRUST5.1 (Mooney et al., 1998) respectively. Compressional wave velocity $v_p$ and density $\rho$ variations are obtained by scaling the shear wave velocity $v_s$ and model S20RTS: $\delta v_s/v_p$ is obtained by scaling $\delta v_s/v_s$ by a factor of 0.5, and $\delta \rho/\rho$ is obtained by scaling $\delta v_s/v_s$ by a factor of 0.3, the scaling factors being those used in making the S20RTS model. Overall, mantle heterogeneity corrections are found to be small compared to measured coefficient size for both $c_{\alpha}$ and $d_{\alpha}$. This indicates that the main contribution to these zonal elastic coefficients is from inner core structure; the same cannot be said of the non-zonal coefficients ($l \neq 0$), which are, however, not used in the present work.

Likewise, the anelastic splitting function coefficients $d_{\alpha}$ and $d_{\alpha}$ are corrected for upper mantle attenuation heterogeneity as described by the model QRFSS12 (Dalton et al., 2008). This shear attenuation $q_{\mu}$ model spans the upper mantle in the depth range 24.4–650 km. For the purpose of the present corrections, the model is not extrapolated to the lower mantle, whose lateral attenuation structure remains poorly understood. Furthermore, variations of bulk attenuation $q_{\alpha}$ are set to zero; this choice is found to have no significant effect on the mantle attenuation corrections. Crustal corrections to $d_{\alpha}$ are not applied. We note that mantle attenuation corrections to $d_{\alpha}$ are negative for all modes, whereas mantle velocity and density corrections to $c_{\alpha}$ are positive. This is expected since upper mantle shear wave velocity and attenuation structures are known to be anticorrelated (Dalton et al., 2009): colder regions are both faster and less attenuating than the global average. Though the present $d_{\alpha}$ mantle correction only takes into account attenuation structure in the upper mantle, it seems unlikely that unmodelled attenuation variations in the lower mantle could completely explain the large $d_{\alpha}$ coefficients of the inner core sensitive modes employed here. Considering the whole mantle, either the mantle attenuation predictions would retain their negative sign with depth, increasing the size of the corrected $d_{\alpha}$ of inner core sensitive modes, or the sign could flip,
Fig. 1. The sensitivity kernels $K_{a}$, $K_{b}$, $K_{c}$, for the 20 self-coupled modes used to build our anisotropy models. ICB stands for the inner core boundary; these kernels only exist within the inner core. $K_{a}$ = red line, $K_{b}$ = blue line, $K_{c}$ = green line. The kernels are plotted separately for Earth structure angular orders $s = 2$ and $s = 4$; note that the modes $1 S_{1}$ and $20 S_{1}$ have no $s = 4$ sensitivity. Only some of the $s = 4$ sensitivity shown here is exploited, depending on whether the measured $c_{40}$ and $d_{40}$ of a particular mode are trusted or not (see text). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

decreasing the size of the mantle corrections applied to the measurements.

Overall, it is found that the mantle corrections applied to either the elastic or the anelastic splitting function coefficients do not change the inversion results to any significant degree.

We also note that in our previous attempts to measure the anelastic splitting function coefficients for normal modes that are sensitive to structure in the Earth’s mantle and crust, but not in the inner core, we have been unable to resolve any non-zero $d_{20}$ or $d_{40}$ coefficients (Mäkinen and Deuss, 2013b). As these mantle mode measurements were robust, we then concluded that the observation of non-zero anelastic zonal splitting function coefficients was restricted to modes sensitive to the Earth’s inner core and therefore a consequence of structure in the inner core. This observation is in agreement with the mantle corrections applicable to the $d_{0}$ and $d_{40}$ coefficients being small, as discussed above. Furthermore, these mantle mode measurements lend further credibility to our inference that structure in the lower mantle, which is not addressed by the QRF512 calculated coefficient corrections, is not a likely contributor to the measured $d_{40}$ coefficients. Similar to the non-zonal elastic splitting function coefficients, the non-zenal anelastic splitting function coefficients $d_{a}(t 
eq 0)$, which are not considered here, could plausibly contain contributions attributable to mantle attenuation variations.

4. Results

4.1. Velocity and attenuation anisotropy models

Fig. 2 shows our inner core velocity and attenuation anisotropy models, in the form of the radial dependence of the anisotropy parameters $\alpha$, $\beta$, $\gamma$ (Fig. 2a) and $\alpha^I$, $\beta^I$, $\gamma^I$ (Fig. 2b). Our models of velocity (Fig. 2a) and attenuation (Fig. 2b) anisotropy parameters show that compressional anisotropy, the dominant quantity in terms of normal mode sensitivity, is positive, both for velocity ($\alpha$) and attenuation ($\alpha^I$).

As shown in Fig. 2a, we best fit the elastic $c_{40}$ data with a velocity anisotropy model that has two radial basis functions, $r^{0}$ and $r^{2}$. This choice of radial parametrization is of particular importance when fitting the $c_{40}$ coefficients. For this velocity anisotropy model, as well as the attenuation anisotropy model, the cross-validation test results, indicating parameter variability, are shown with dashed lines. The model parameters $\alpha$, $\beta$ and $\gamma$ obtained using two different depth parametrizations, $a$ and $a + br^{2}$ (variance reduction 0.78 for the latter), are given in Table 1. It is clear from Table 1 that the volumetrically averaged values of $\alpha$ and $\beta$, obtained in the two inversions of the $c_{30}$ and $c_{40}$, are in good agreement; the values of $\gamma$ differ somewhat, but this is within the limits of variation found in other authors' models (Table 1). The $a + br^{2}$ radially parametrized model is chosen as the final
model since the $a$ parametrized model is completely unable to reproduce the larger of the $C_{40}$ coefficients, predicting their values close to zero, in clear disagreement with the measurements. This indicates that the $C_{40}$ coefficients are sensitive to not only the volumetrically averaged values of the anisotropy parameters — which are similar between the two elastic models — but also the depth trends of these parameters.

In contrast to the velocity anisotropy inversion, our attenuation anisotropy inversion, using anelastic $d_{a0}$ data, is not stable with respect to removing individual modes if the depth parametrization $a + br^2$ is used. The simpler depth parametrization $a$, on the other hand, yields robust results; therefore our preferred model for attenuation anisotropy is radially constant (variance reduction 0.55). This choice of a simple yet stable model is deliberate: we would rather be certain of resolving the average attenuation anisotropy features than produce models that appear more detailed but lack robustness. We attribute the instability of the $a + br^2$ parametrized $d_{a0}$ inversion, which requires twice the number of parameters to be found compared to the inversion with $b = 0$, to limited numbers of data: we only have 7 $d_{a0}$ coefficients, whereas the $C_{40}$ inversion benefited from 14 $C_{40}$. The final attenuation anisotropy model parameters $a^2$, $b^2$, and $y^2$ are also given in Table 1.

We then use Eqs. (5)–(10) and the volume averaged values of the anisotropy parameters of Fig. 2, given in Table 1, to calculate the variations of velocity $v$ and attenuation $q$ as a function of wave direction $\xi$ in the inner core. The resulting anisotropy curves are shown in Fig. 3. We find that, for compressional waves, polar directions ($\xi = 0^\circ$) are both faster and more attenuating than those in the equatorial plane ($\xi = 90^\circ$). This result concurs with previous body wave observations, namely that compressional wave attenuation anisotropy correlates with velocity anisotropy (Souriau and Romanowicz, 1996; Niu and Wen, 2002; Cormier and Li, 2002; Yu and Wen, 2006).

As an important test of the validity of our final anisotropy models, we test how well they can recover the input splitting function coefficients. This comparison is displayed in Fig. 4. As is evident in Fig. 4a, our final velocity anisotropy model can recover both the $C_{20}$ and $C_{40}$ relatively fairly, reproducing both sign and magnitude in most cases.

Concerning the recovery of $d_{20}$ and $d_{40}$ using our constant attenuation anisotropy model, we note that the model does recover the positive $d_{20}$ (Fig. 4b). However, we cannot explain the negative $d_{a0}$, in particular the negative $d_{20}$ (for modes $5S$, $3S$, and $5S$; these coefficients have been included in the modelling). The discrepancy between these modes and the remaining 17 positive $d_{20}$ coefficients is likely to result from effects that are not included in the present modelling. One such effect could comprise deviations from North–South symmetric cylindrical attenuation anisotropy of the inner core. This in turn might take the form of a cylindrical symmetry whose axis does not coincide with the Earth’s rotation axis; for such an anisotropy, non-zero non-zonal $d_{n0}$, $t \neq 0$, would be introduced in addition to non-zero $d_{a0}$, and the values of $d_{a0}$ would also change (see the sections ‘Tilted symmetry axis’ by Tromp, 1995 and Mäkinen and Deuss, 2013a). Attenuation anisotropy with symmetry lower than the transverse isotropy considered here is also a possible explanation for the dis-

### Table 1

<table>
<thead>
<tr>
<th>Inversion</th>
<th>$(a^2)$</th>
<th>$(b^2)$</th>
<th>$(y^2)$</th>
<th>$(a^2)$</th>
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<td>$a$</td>
<td>2.99</td>
<td>0.72</td>
<td>0.15</td>
<td>5.582</td>
<td>−0.006</td>
<td>2.686</td>
</tr>
<tr>
<td>$a + br^2$</td>
<td>−0.74</td>
<td>0.15</td>
<td>0.32</td>
<td>−0.881</td>
<td>−0.404</td>
<td>−0.759</td>
</tr>
<tr>
<td>Woodhouse et al. (1985), $a$</td>
<td>0.18</td>
<td>0.43</td>
<td>0.09</td>
<td>−0.39</td>
<td>−0.09</td>
<td>−0.21</td>
</tr>
<tr>
<td>Woodhouse et al. (1985), $br^2$</td>
<td>3.46</td>
<td>0.85</td>
<td>−0.54</td>
<td>2.27</td>
<td>1.04</td>
<td>−0.52</td>
</tr>
<tr>
<td>Tromp (1993)</td>
<td>6.24</td>
<td>1.14</td>
<td>−1.98</td>
<td>5.0</td>
<td>0.8</td>
<td>−0.48</td>
</tr>
<tr>
<td>Durek and Romanowicz (1999)</td>
<td>5.0</td>
<td>0.8</td>
<td>−0.5</td>
<td>1.0</td>
<td>0.8</td>
<td>−0.5</td>
</tr>
<tr>
<td>Ishii et al. (2002)</td>
<td>3.49</td>
<td>0.988</td>
<td>0.811</td>
<td>3.49</td>
<td>0.988</td>
<td>0.811</td>
</tr>
<tr>
<td>Beghein and Trampert (2003)</td>
<td>5.87</td>
<td>1.71</td>
<td>1.53</td>
<td>5.87</td>
<td>1.71</td>
<td>1.53</td>
</tr>
</tbody>
</table>

Fig. 2. Radial models of inner core anisotropy. a) velocity anisotropy, b) attenuation anisotropy. Dashed lines indicate the results of cross-validation runs performed with ten per cent of the modes omitted each time. The volume average values for the anisotropy parameters are: $a^2 = 5.582 \times 10^{-2}$, $b^2 = 0.006 \times 10^{-2}$, and $y^2 = 2.686 \times 10^{-2}$, in fraction of inverse compressional attenuation, for attenuation, and $a^2 = 3.46$, $b^2 = 0.85$, and $y^2 = −0.54$, in per cent, for velocity. (For interpretation of colours in this figure, the reader is referred to the web version of this article.)
Fig. 3. Volume averaged inner core anisotropy as function of ray angle, plotted for comparison with body wave and mineral physics models for set values of ξ, a) of compressional velocity v_p, b) of compressional attenuation q_p, c) of shear velocity v_s, d) of shear attenuation q_s. For shear, δq_{seq} and δv_{seq} are for shear waves polarized in the equatorial plane; δq_{sme} and δv_{sme} are for shear waves polarized in the meridional plane. The angle ξ is the angle between the North–South axis and the ray path in the inner core at its turning point; ξ = 0° corresponds to the polar direction and ξ = 90° to any direction in the equatorial plane. The shaded areas show model confidence levels from cross-validation. The grey line a) indicates hemisphere-averaged body wave results of Irving and Deuss (2011). (For interpretation of colours in this figure, the reader is referred to the web version of this article.)

Fig. 4. Observed and predicted splitting function coefficients for the anisotropy inversions. a) Elastic splitting function coefficients c_{ij}, used for velocity anisotropy, b) anelastic splitting function coefficients d_{ij}, used for attenuation anisotropy. Degree 2 coefficients are indicated by black circles and degree 4 coefficients by red circles. Black and red triangles mark the degree 2 and degree 4 coefficients, respectively, of the mode 27 S2, whose splitting functions are shown in Fig. 5. The diagonal line indicates perfect agreement between observed and predicted coefficients. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

crepancies, although the theory linking anelastic splitting function coefficients to parametrizations of such anisotropy is, at present, unresolved. Inner core anelastic heterogeneities could also contribute. Owing to the small number of data available for the inversion, parametrizations more complicated than cylindrically symmetric anisotropy cannot at present be accommodated, and thus the negative d_{20} must remain unexplained. Furthermore, it is evident in both Fig. 4b, and the variance reduction of 0.55 associated with the anelastic splitting function coefficient agreement, that the attenuation anisotropy model does not explain the observed splitting range in full, as perhaps expected given the simplicity of this model. However, the overall recovery of d_{s0} coefficients using the

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attenuation anisotropy model, shown in Fig. 4b, is substantially improved compared to the null hypothesis: under the null hypothesis (no inner core cylindrically symmetric attenuation anisotropy), all $d_{20}$ and $d_{40}$ would be predicted to be zero. Furthermore, given the definition of the variance reduction in Eqs. (15)–(17), the VR for the null hypothesis would be 0, a value which both the elastic and the anelastic models substantially exceed.

We further illustrate the success of our relatively simple anisotropy models in predicting splitting function coefficients by considering the mode $\gamma S_2$, whose splitting functions are shown in Fig. 5. For $\gamma S_2$, as for most other inner core sensitive modes, the zonal splitting is observed as strong positive anomalies near the poles and a negative anomaly along the equator. These are present in the observed elastic (Fig. 5a) and anelastic (Fig. 5b) splitting functions, and reproduced by the predictions from our velocity (Fig. 5c) and attenuation (Fig. 5d) anisotropy models, respectively. Contributions expected from mantle structure to both the elastic and the anelastic splitting functions are found to be so small as to not be observable in Fig. 5.

4.2. Reconciling normal mode models with body wave results and hemispherical variations

Comparison of our normal mode models with body wave observations reveals that the anisotropy of compressional velocity $v_p$ we resolve (Fig. 3a) is in the same direction as, but smaller than, body wave estimates, which currently require 3–4 per cent velocity anisotropy (see e.g. Morelli et al., 1986; Irving and Deuss, 2011, and references therein). This discrepancy between normal mode and body wave studies is well known, at least in the case of velocity anisotropy (Ishii et al., 2002). It has been attributed to global averaging inherent in normal mode studies, as well as to biased sampling in body wave analyses. Both of these factors are discussed in detail below.

As for attenuation, fewer studies exist. Body wave studies typically find an equatorial compressional quality factor ($Q_p$, defined as $1/q_p$) of 600 and a corresponding polar quality factor in the range 200–420 in the upper inner core (Creager, 1992; Yu and Wen, 2006), the part of the inner core whose attenuation is best described in existing body wave work. For comparison, for a given equatorial quality factor of 600, the value of $\alpha$ in our model predicts a polar quality factor of 138. This is smaller than the body wave estimates and indeed difficult to reconcile with the body wave values for realistic average inner core attenuation, indicating that the compressional attenuation anisotropy we find is stronger.

Hemispherical variations in inner core velocity anisotropy are a well-established feature, and have been observed using both body waves (Tanaka and Hamaguchi, 1997; Niu and Wen, 2001; Yu and Wen, 2007) and normal modes (Deuss et al., 2010). Furthermore, some body wave studies (Niu and Wen, 2001; Li and Cormier, 2002; Cormier and Li, 2002; Cormier, 2007) also find that inner core attenuation anisotropy varies between the hemispheres. The western hemisphere is found to exhibit larger velocity and attenuation anisotropy than the nearly isotropic eastern hemisphere.

Isolated normal modes in the self-coupling approximation, such as those considered here, are sensitive to the global average of the two hemispheres. Thus the velocity and attenuation anisotropy, predominantly found in the western hemisphere, would appear smeared over the entire inner core when observed using normal modes using the self-coupling approximation. This complicates comparisons between normal mode and body wave anisotropy models: for a hypothetical inner core, divided exactly in half between an anisotropic western hemisphere and an isotropic eastern hemisphere, the globally averaged anisotropy values of our normal mode models must be multiplied by a factor of two in order to obtain the magnitude of anisotropy in the western hemisphere. Should the western hemisphere occupy a different proportion of the inner core (estimates range from 240 to 165° (Tanaka and Hamaguchi, 1997; Creager, 1999; Niu and Wen, 2001; Oreshin and Vinnik, 2004; Irving and Deuss, 2011)), the multiplication factor must be adjusted accordingly.
A further complication arises from uneven sampling of the inner core by body waves: for example, in a recent study, Irving and Deuss (2011) find a whole inner core compressional wave velocity anisotropy of 3.8 per cent, and, for a western hemisphere of 165°, corresponding western hemisphere only value of 4.8 per cent. Calculating the global average from the values for the two hemispheres yields 2.5 per cent compressional velocity anisotropy, showing that the whole inner core value of 3.8 per cent is biased by sampling the western hemisphere more. This discrepancy between global and hemisphere values is likely to result from difficulty in determining the sizes of the two hemispheres, as the suggested hemisphere boundaries, in particular the Pacific one, may be poorly sampled by raypaths.

Some body wave studies (Yu and Wen, 2006) also report observations of hemispherical variation effects of compressional attenuation anisotropy; at present, it is not clear whether normal modes require this. To establish whether normal modes resolve hemispherical differences in attenuation anisotropy, one would need to robustly measure at least zonal (l = 0) odd-degree anelastic splitting function coefficients \( d_{ls} (s = 1, 3, 5) \) of inner core sensitive modes. These measurements require the cross-coupling approximation, and are beyond the scope of the data presently available in terms of measurement resolution.

Taking these complications into account, precisely reconciling normal mode and body wave numerical values is not presently possible. The sense of the velocity and attenuation anisotropy resolved using the two types of data would, however, be expected to be in agreement, which is observed to be the case.

5. Physical interpretation of anisotropy models

Attenuation of seismic waves in a crystalline material can be caused by either single or multiple scattering of the energy from interfaces within the material, or by intrinsic (viscoelastic) energy loss due to atomic-level changes of order within the material, excited by the passage of the seismic energy. Within the Earth’s inner core, normal modes have wavelengths several orders of magnitude longer than any estimate of inner core grain size. Therefore normal modes in the inner core are not affected by grain boundary scattering, and are thus only attenuated by intrinsic mechanisms.

Our models then do not constrain the scattering component of attenuation. This component could still be anisotropic, as argued in body wave studies (Cormier and Li, 2002). It is clear, however, that the attenuation anisotropy quantified here cannot originate from anisotropic scattering, the currently proposed explanation (Cormier and Li, 2002) for compressional body wave attenuation anisotropy. Instead, an alternative explanation is required for the observed inner core attenuation anisotropy. We rule out the presence of melt as the origin of the anisotropy, as this would lead to directions of low compressional velocity being more, not less, attenuating (Singh et al., 2000). The attenuation anisotropy here must then reflect the anisotropy of intrinsic (viscoelastic) attenuation of the crystalline inner core material.

Anelasticity due to grain boundary relaxation is one possible source of attenuation in the inner core. However, current estimates (Bergman, 1998; Calvet and Margerin, 2008; Mommersteeg et al., 2010) of inner core grain size suggest grains larger than at least 10–100 m, making the grain boundary density vanishingly small and thus reducing grain boundary relaxation to an insignificant contribution. Furthermore, the effects of pressure on grain boundary dynamics are expected to reduce the strength of such relaxation even further, rendering grain boundary relaxation an unlikely explanation for the observed intrinsic attenuation anisotropy. Dislocation-related relaxations provide another potential mechanism of anelastic loss (Nowick and Berry, 1972). Observed as the “Bordoni peak” in hexagonal metals at room temperature and pressure, this phenomenon is seen in cold-worked metals that have high dislocation densities. The Bordoni peak disappears in metals that have been annealed. The high homologous temperature of the inner core, combined with the lack of any clear origin for deformation forces in a slowly-growing inner core, lead to the supposition that dislocation densities are likely very low. The Bordoni mechanism, therefore, appears an unlikely origin for the observed attenuation.

Alternatively, anisotropic Zener strain relaxation effects are known to exist in metallic alloys, including hcp alloys, which have a low concentration of solute, such as light elements (LeClaire and Lomer, 1954; Seraphim and Nowick, 1961; Berry, 1961; Kappesser et al., 1997). These relaxation effects, which contribute to intrinsic attenuation, occur due to orientational realignment of pairs of typically substitutional solute atoms when compressional or shear stress is applied (Seraphim and Nowick, 1961; Berry, 1961). Such reorientations, illustrated in Fig. 6, show anisotropic distribution by virtue of the inherent elastic anisotropy of the host crystal structure as well as the anisotropy of solute pair distribution within the host structure. This leads to anisotropic attenuation as an intrinsic property of the solid. While experimental study of the intrinsic anelastic attenuation of materials at the conditions of Earth’s inner core remains extremely challenging, recent computational studies of the dynamics of iron in the inner core have highlighted the importance of such relaxations (Belonoshko et al., 2010), supporting the supposition that anelasticity associated with the Zener effect may play a significant role in the mechanical response of the core.

Furthermore, Zener relaxation would be expected to show frequency dispersion, so variations in observed anelasticity may be expected between body wave and normal mode studies, whose frequency ranges differ by three orders of magnitude. Whilst the Zener effect is expected to show a frequency dependency, the particular dispersion characteristics of this dependence remain uncertain. Nonetheless, general attenuation behaviour can vary significantly with pressure and temperature for Debye-like relaxation processes, and at high homologous temperatures distributions of relaxation times are likely to be broad, increasing the potential significance of such anelastic effects (see e.g. Carpenter and Zhang, 2011). Further experimental studies, especially at high tempera-
tures and pressures, and using hexagonal metals, would be useful for probing this issue further.

That intrinsic inner core attenuation anisotropy be explained by anisotropic Zener relaxation effects requires that enough sub-
stitutional light elements be present in the inner core, such that defect pairs occur with sufficient probability. Experiments have shown that pair reorientation becomes measurably significant in alloys with substitutions of as little as two atomic per cent (at.%), with the magnitude of anelastic attenuation scaling as the square of the solute concentration (Sarafim and Nowick, 1961). Moreover, the commonly considered inner core light elements Si and O are known to be substitutional (Affé et al., 2002), incorporated onto Fe-sites in hcp-Fe, now considered the most likely iron crystal structure under inner core conditions (Stixrude and Cohen, 1995; VoCadlo et al., 2000, 2003; Nguyen and Holmes, 2004; Tateno et al., 2010). Thus the existence of inner core intrinsic attenuation anisotropy by Zener relaxation is entirely consistent with estimates of as much as 5 at.% Si and O in bulk iron (Badro et al., 2007).

We also postulate that future observation of possible depth vari-
ations in intrinsic attenuation anisotropy could be used to probe the changes of inner core light element concentration and compos-
ition with depth, though our current anelastic splitting function coefficient data restrict our attenuation anisotropy model to be depth-invariant.

6. Conclusions

We have inverted the anelastic and elastic splitting function coefficients $d_{20}$, $d_{40}$ and $c_{20}$, $c_{40}$ of twenty inner core sensitive normal modes for cylindrically symmetric models of attenuation and velocity anisotropy of the Earth’s inner core. Our models show that both compressional and shear velocity and attenuation in the Earth’s inner core are anisotropic. Compressional waves traversing the inner core in the polar symmetry direction are both faster and more strongly attenuated than those propagating in the equatorial plane.

As our attenuation model is constructed using normal modes, whose long wavelengths make them transparent to scattering atten-
tuation in the inner core for any realistic estimates of inner core grain size, the attenuation anisotropy we have observed necessar-
ily reflects the anisotropy of intrinsic attenuation in the inner core. This intrinsic attenuation anisotropy can be interpreted in terms of anisotropic Zener relaxation in the metallic alloy comprising the inner core. This mechanism can only take hold if sufficient light element atoms, as solute, are present in the inner core. Thus our observation of anisotropic attenuation in the inner core is consist-
ent with the presence of a few atomic per cent of light elements in the inner core, independent of any considerations of inner core density.

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